COMPETITIVE PRICING AND SOURCING PROBLEMS IN

OPERATIONS MANAGEMENT

by

Varun Gupta

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Dedicated to my parents, family, and teachers.

COMPETITIVE PRICING AND SOURCING PROBLEMS IN OPERATIONS MANAGEMENT

by

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This dissertation addresses three important problems concerning competitive pricing and sourcing in operations management.

The first problem focuses on a new choice model based on willingness-to-pay (WTP), incorporating sequential decision making of customers. We compare WTP-choice model with the commonly used Logit model. Using WTP-choice model, we compare equilibrium prices, demands and profits of several contexts: without considering inventory and with stockouts – lost sales and backorders for "retailer favoring" customers and for "availability favoring" customers. An interesting result with the WTP-choice model is the "loose coupling" of retailers; prices are not coupled but profits are. We show that competition between retailers with dependent WTPs can cause price cycles under some conditions. We consider real-life data on sales of yogurt, ketchup, candy melt, and tuna, and check if WTP-choice model, standard or mixed Logit model fits better and predict the sales better.

The second problem analyzes the issue of contingent dual sourcing under supply chain disruption and competition. We consider a supply chain in which two suppliers sell components to two competing manufacturers producing and selling substitutable products. Supplier U is unreliable and cheap while Supplier R is reliable and expensive. Firm C uses a contingent dual sourcing strategy (CDSS) and Firm S uses a single sourcing strategy (SSS). We show that supply disruption and procurement times jointly impact the firms' buying decisions, and through numerical computations, we obtain additional managerial insights. Finally, as extensions, we study the impact of endogenizing equilibrium sourcing strategies for asymmetric and symmetric firms, and of capacity reservation by Firm C with Supplier R to mitigate disruption.

The third problem investigates a market where distributors compete to sell experiential products (e.g., movies and music), should they use pay-per-unit pricing or use subscription pricing? We study market dynamics when a content provider is selling to two distributors which are using different pricing modalities and characterize conditions where both can coexist profitably, where only one can make money and where price wars would be expected. We also investigate the role of the contractual powers of the players.

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CHAPTER 1

A WTP-CHOICE MODEL: EMPIRICAL VALIDATION AND COMPETITIVE PRICING WITH STOCKOUTS

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1.1. Introduction

Willingness To Pay (WTP) is the maximum amount a customer would be willing to pay in order to receive a product and plays a central role in the selection of a product from several choices. The primary aim of this study is to propose a discrete choice model based on WTPs and to apply the model to pricing. The proposed WTP based choice model is used to study competitive pricing, first by focusing on dependencies among WTPs and customer preferences and by ignoring inventory consideration, and then under stockouts that alter customer choices. Another aim is to check the efficacy of the WTP-choice model by comparing it with the commonly used (multinomial) Logit model and mixed Logit model in terms of the log-likelihood values as well as the accuracy of choice estimates. Comparisons involve real-life data on candy melts, yogurt, ketchup and tuna sold by different retailers (firms) in different markets.

The classical approach to customer choices is through Logit models (e.g., McFadden 1980). Logit model and its extensions have so far been the preferred model (Chandukala et al. 2008 and pp.78-85 Schroeder 2010). The customer choice literature is growing with the exploration of the process of forming perceptions and beliefs in different practical contexts. Figure 1.1 shows the choice process for a customer given his experiences and information. The perceptions and preferences of a customer shaped from his memories and knowledge of products as well as the prices lead to the product choice. As opposed to Figure 1.1, McFadden (2001) connects a customer's memory/knowledge to the decision process with a single path by combining his perception/belief and preferences. In this paper, we separate a customer's perception/belief and preferences using the lower path in Figure 1.1. Through this separation, we explicitly represent customer preferences. Therefore, in line with the direct utility approach (Chintagunta and Nair 2011) to customer choice modelling, WTP-choice model directly incorporates WTPs and customer preferences, and provides direct insights into customer behavior.

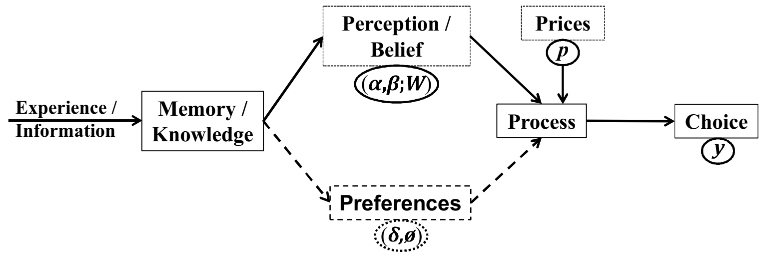


Figure 1.1. A choice process with its inputs and output.

The premise behind modelling preferences includes capturing a customer's established habits, routine, convenience of shopping in certain patterns or sequences and relative magnitude of search/transportation costs for retailers/products. Dillon et al. (2013) study two Chicago based grocery stores: Jewel and Dominick's. They find out that respectively 53% and 41% of shoppers are likely to first visit Jewel and Dominick's, so shoppers exhibit habits. These habits are collectively called preferences and can be independent of product prices. Preferences can also be deployed as measures of customer loyalty (Bijmolt et al. 2010). One can argue that the parameters of Logit model capture choice probabilities, which may *indirectly* be interpreted as (revealed) preferences. Without resorting to such an interpretation, we start with (stated) preferences *directly* incorporated into WTP-choice model.

Product prices and utility (quality) vary across retail stores and over time. Most choice models such as Logit model assume that the customers are aware of the prices and utilities for all products. In reality, customers are unlikely to maintain such extensive information especially for high-purchase frequency items, even if they are willing to spend the cognitive effort for objectively processing this information, which may lead to a state of paralysis-by-analysis. Instead, each customer gathers knowledge about some, but perhaps not all, products to make a choice, e.g., Seiler (2010) maintains that customers infrequently check product prices and quality at a few retail stores. Preferences help a customer make a choice before checking all products (Carlson et al. 2009). Hauser and Wernerfelt (1990) show that customers have *consideration sets* whose size range from 2 to 8 depending on product categories. Hauser (2014) reviews heuristic rules for first forming a consideration set and then choosing a product from this set. WTP-choice model explicitly models customers' consideration set, as well as the consideration set heterogeneity among the customers. It uses preferences to explicitly rank the products within a particular consideration set. It hence captures the sequential search for a product, which usually ends before considering all products.

As decision rules, customers use two-stage decision process and threshold screening to simplify complicated decisions (Gilbride and Allenby 2004). A customer may be satisfied with a reasonable product and stop searching for better products (Stüttgen et al. 2012) when prices are unavailable or costly to find out. Such a customer does not necessarily maximize his surplus – difference between the utility obtained from a product and its price. Bounded rationality (Gigerenzer and Selten 2002) of customers can be used to explain why customers do not always maximize their surplus and instead are satisfied with just nonnegative (feasible) surplus. Even extremely rational customers may not care about tedious surplus maximization when buying low-value items, whereas they can be quite meticulous when buying high-value items. It is better to assess the appropriateness of a customer's search for maximum vs. feasible surplus after specifying the product and its value. Hence, the domain of choice models is very broad and can accommodate new models, especially those that are simple, based on customer preferences and a feasible surplus criterion.

In Logit model, customer n chooses the product i from M products at a price p^i with the probability

$$\lambda_n^i = \exp(\alpha_i + \beta p_n^i)\Big[\sum_{j=0}^{M} \exp(\alpha_j + \beta p_n^j)\Big]^{-1}. \tag{1.1}$$

The choice of $i = 0$ indicates *no-purchase* with $p_n^0 = 0$. The term $\exp(\alpha_i + \beta p_n^i)$ is the attractiveness of option i and its increase/decrease in price p_n^i is governed by parameters $\alpha_0, \alpha_1, \ldots, \alpha_M$ and β (pp.491-495 Cameron and Trivedi 2005). Each customer maximizes the surplus and each utility has a double-exponential distribution (p.306 Talluri and van

Ryzin 2004). Recently, Farias et al. (2013) and Jagabathula and Rusmevichientong (2013) propose nonparametric choice models consistent with Logit model and then predict revenues respectively for an automaker and a television retailer.

To better appreciate the differences between WTP-choice model and Logit model, we provide an example of a customer who considers buying organic yogurt or regular yogurt, and prefers organic yogurt. In real-life, the choice of this organic yogurt preferring customer may not be affected by the price of regular yogurt. This is because such a customer buys the organic yogurt if it is affordable. Basing the choice only on the affordability of the preferred product corresponds to the satisficing criterion of WTP-choice model. On the contrary, this choice, if captured by Logit model, is affected by the price of regular yogurt due to surplus maximization over organic and regular yogurts.

Many items at retailers experience stockouts (DeHoratius et al. 2008), so the impact of a stockout on the choice process is important to investigate. Choice models such as Logit model or location choice model cannot explicitly capture the effects of a stockout on the customer behavior (Gaur and Honhon 2006). To capture these, Musalem et al. (2010) use a modified Logit model. Mahajan and van Ryzin (2001) take stockout events into account by dynamically removing the stocked-out items from the customer's consideration set. Kök and Fisher (2007) highlight the inability of the standard Logit model to capture the impact of stockouts on customer behavior, develop a demand rate function to capture this behavior and use it for demand estimation and assortment optimization. However, these works do not study pricing decisions.

Choice models are the building blocks of price optimization (Özer and Phillips 2012). Alptekinoğlu and Semple (2013) propose an exponomial choice model and compare price optimization results obtained by using exponomial choice model with those obtained by using Logit model. Both exponomial and Logit frameworks lead to non-trivial price optimization (Li and Huh 2011). When customers do not maximize their utility as in the case of low-price items (Stüttgen et al. 2012) or when the utility distribution does not follow the double-exponential distribution as opposed to Logit model (or the normal distribution as in Probit model), one needs a new choice model, possibly based on WTPs.

WTP estimation has received significant attention and uses scanner or survey data (Wertenbroch and Skiera 2002). In the scanner data methods, there are buyers and non-buyers. WTP of a buyer is at least the price being offered and that of a non-buyer is less than the price being offered. Earlier studies assumed WTP to be a single price point in customer's propensity to buy, however later studies consider it to be a range (Wang et al. 2007). WTP can be indirectly constructed by starting with a utility framework, however, estimating it directly fits the data better in general, decreasing the chances of exceedingly large estimated WTP variances (Scarpa et al. 2012). Our chapter parametrically estimates WTP distributions using a likelihood criterion. It should be noted that WTP does not have to be parameterized for WTP-choice model; only in estimation and sharpening some results, we resort to parameterization.

We compare Logit model and WTP-choice model and show that competitive pricing with WTP-choice model is relatively easy to analyze and implement. In particular with WTP-choice model, the prices are "loosely coupled"; each retailer should charge monopoly prices in competition as these constitute the equilibrium, but that retailer's profit depends on prices of all the retailers. At the onset, loose coupling seems to be surprising, this however relates to *monopolistic competition*, where "each firm . . . can ignore its impact on, and hence reactions from, other firms" (p.529 Hart 1985). Monopolistic competition in a market is due to the presence of customers who differentiate between the brands in the market but do not easily switch to another brand due to slight changes in the price of a brand. This description of customers who exhibit a friction to brand switching hints at price independent customer preferences. However, we note that these preferences are not sufficient for loose coupling, which vanishes with dependent WTPs. Independence of WTPs and the independence of preferences from prices together drive loose coupling and can yield closed-form expressions for equilibrium prices.

We illustrate with an example that dependence of WTPs can cause *price cycles*, where prices charged by the retailers alternate within a set of prices. We provide conditions to rule out price cycles and to conclude the presence of an equilibrium with a single price for each retailer. Although price cycles are not usually considered as a solution to a pricing game in Operations Management, they are empirically observed (Noel 2008) and theoretically explained (Maskin and Tirole 1998). Dependence of WTPs in our model can provide another explanation for these cycles.

We extend WTP-choice model to study competitive pricing under substitutions driven by stockouts. Substitutions in our model are based on stockout probabilities rather than stockout events. Compared to stockout events, stockout probabilities are more stable in the sense that a probability is the average frequency of many events. This stability makes stockout probabilities for customers easier to obtain and use in a choice model (Hopp and Xu 2008).

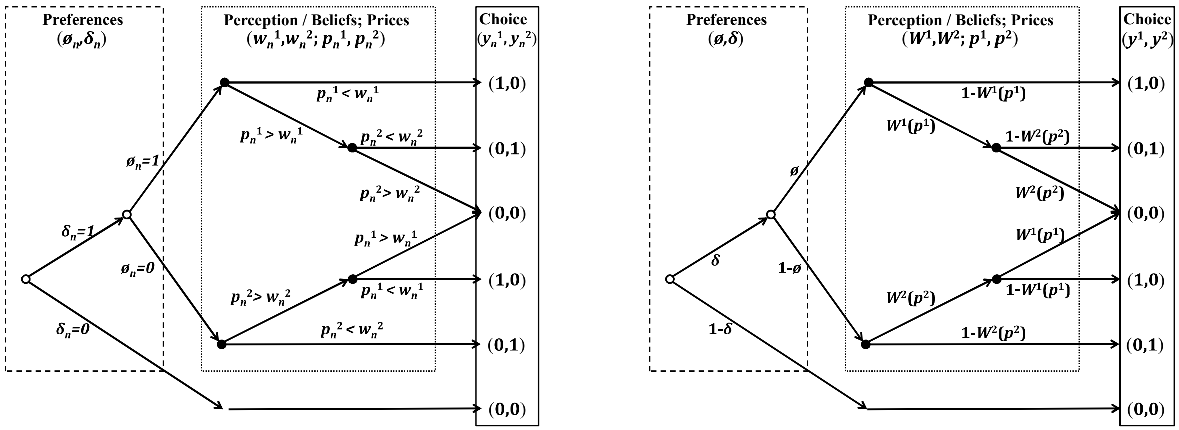
We study lost sales and backorders for two types of customers – retailer and availability favoring. A retailer favoring customer backorders from his preferred retailer if this retailer is stocked out, whereas an availability favoring customer backorders from his preferred retailer if all the retailers are stocked out. Otherwise, an availability favoring customer buys from a retailer with available inventory. A distinction is usually not made between retailer and availability favoring customers, partly because such a model requires adopting a sequential decision framework for customer choices, which are not common in the literature. A related study is the survey of more than 71,000 customers reported in Corsten and Gruen (2004): upon a stockout at a retailer, 9% of customers do not purchase, 15% delay purchase, 45% substitute a product (different or same brand), 31% buy at another retailer. Although these do not map one-to-one with lost sales, backorders, retailer or availability favoring behavior, they indicate the presence of these behaviors and possibility of measuring them through a survey. By modelling backorders with retailer favoring customers or backorders with availability favoring customers, we illustrate the versatility of WTP-choice model and test the robustness of loose coupling property. Provided that WTPs are independent of each other and preferences are independent of prices, this property continues to hold in all of the stockout extensions of WTP-choice model. We compare the equilibrium sales probabilities and profits under retailer and availability customers, and discuss the effectiveness of loyalty programs in the presence of these type of customers.

Choice models map prices to choices through parameters that need to be estimated (Olivares et al. 2008 and Akşin et al. 2013). Although the proposed WTP-choice model is non-parametric, we parameterize the WTP distribution to simplify the estimation scheme, which uses likelihood maximization. We assess the estimation and prediction efficacy of WTP-choice models in comparison with Logit models using real-life data on yogurt, ketchup, candy-melt, and tuna. The estimation and prediction results show that WTP-choice models compare well with Logit models.

This chapter's contributions include introducing WTP-choice model in section 1.2, establishing a loose coupling property for competing retailers under independent WTPs and studying equilibrium prices and price cycles under dependent WTPs in section 1.3, extending the loose coupling property to inventory models with lost sales and backorders in section 1.4 and by empirically validating WTP-choice model in section 1.5.

1.2. WTP-Choice Model

WTP-choice model is designed to incorporate WTPs and customer preferences into the choice process and it applies to a context of M products in a market. Customer n is offered product $m \leq M$ at the price p_n^m. Subsequently, customer n decides to buy product m ($y_n^m = 1$) or not ($y_n^m = 0$). In WTP-choice model, customer n has the preferences that are part of the



(a) Customer's perspective. (b) Firm's perspective.

Figure 1.2. Customer decision tree in WTP-choice model.

lower path in Figure 1.1. The parameter δ_n captures the preference of customer n to buy a product or nothing. For example, customer n with $\delta_n = 0$ is interested in none of the products.

To explain product preferences and WTP-choice model, we first consider two products, so $M = 2$. The parameter ϕ_n denotes the preference of customer n: if the customer prefers product 1, he has $\phi_n = 1$; otherwise, $\phi_n = 0$. When $\delta_n = 0$, customer n is not in the market to buy product 1 or 2. For example, such a customer may enter a store to buy other products but does not pay attention to products 1 or 2. When $\delta_n = 1$, customer n walks in the store and checks out the prices of products 1 and 2 to decide to buy or not. This customer does not buy a product if he gets a negative surplus from each product, i.e., the price of each product is higher than his WTP for that product. Hence, customer n buys nothing, i.e., $(y_n^1, y_n^2) = (0, 0)$, when he is uninterested or prices are high relative to WTPs. These respectively correspond to the two $(0, 0)$ choices in Figure 1.2(a).

When $(\delta_n, \phi_n) = (1, 1)$, customer n is in the market to buy a product and prefers product 1 over product 2. If this customer's surplus from product 1 is nonnegative, he buys product 1. The customer arrives at this decision without considering product 2. If the surplus from product 1 is negative, then the customer considers product 2. If the surplus from product 2 is nonnegative, he buys it. Otherwise, he buys nothing. The choice process for a customer with $(\delta_n, \phi_n) = (1, 0)$ is symmetric to the process described above and is shown in Figure 1.2(a).

Given prices (p_n^1, p_n^2), preferences (δ_n, ϕ_n) and customer WTPs (w_n^1, w_n^2), choices of customer n are:

- $(y_n^1, y_n^2) = (1, 0)$: Product 1 if $[\delta_n = 1,\ \phi_n = 1,\ p_n^1 \leq w_n^1]$ or $[\delta_n = 1,\ \phi_n = 0,\ p_n^2 > w_n^2,\ p_n^1 \leq w_n^1]$.
- $(y_n^1, y_n^2) = (0, 1)$: Product 2 if $[\delta_n = 1,\ \phi_n = 0,\ p_n^2 \leq w_n^2]$ or $[\delta_n = 1,\ \phi_n = 1,\ p_n^1 > w_n^1,\ p_n^2 \leq w_n^2]$.
- $(y_n^1, y_n^2) = (0, 0)$: None if $[\delta_n = 1,\ p_n^1 > w_n^1,\ p_n^2 > w_n^2]$ or $[\delta_n = 0]$.

A firm often does not know the preferences or WTPs of each customer as it faces a population of customers. This population has preferences $\{\phi_1, \phi_2, \dots\}$ for $\phi_n \in \{0, 1\}$, and the firm can estimate the probability ϕ that a random customer prefers product 1 over 2. Similar to ϕ, we can use δ for the probability that a random customer is interested in a product. Unlike (δ, ϕ), WTPs are not binary variables so the probability associated with them can be represented by cumulative probability distributions W^1 and W^2. Figure 1.2(b) uses probabilities (δ, ϕ, W^1, W^2) to present a single random customer's choice from the firm's perspective. These probabilities do not necessarily imply heterogeneous customers as they can only imply the lack of a firm's knowledge about identical (homogenous) customers.

Given prices, preferences and independent WTPs, the choice probabilities for customer n are:

- $$\rho_n^1 := \mathrm{P}(y_n^1 = 1, y_n^2 = 0) = \delta(1 - W^1(p_n^1))\{(1 - \phi)W^2(p_n^2) + \phi\}, \quad (1.2)$$
- $$\rho_n^2 := \mathrm{P}(y_n^1 = 0, y_n^2 = 1) = \delta(1 - W^2(p_n^2))\{\phi W^1(p_n^1) + (1 - \phi)\}, \quad (1.3)$$

and $\rho_n^0 = \delta W^1(p_n^1)W^2(p_n^2) + 1 - \delta = 1 - \rho_n^1 - \rho_n^2$. The first term on the right-hand side of (1.2) is the probability that the customer is interested in a product. The second term in the parentheses is the probability that the customer is willing to pay at least the price of product 1. The third term in brackets expresses the sum of the probabilities that the customer prefers product 2 but finds it too expensive and that the customer prefers product 1. A similar interpretation can be given for the probability ρ_n^2.

In the M-product version of WTP-choice model, we assume that customers have a collection of ordered consideration sets of $\mathfrak{L}_i$ for $i = 1, \ldots, S$ and each set has size $L \leq M$. Each product belongs to at least one of the consideration sets. The probability that a customer has the consideration set $\mathfrak{L}_i$ is ϕ_i and $\sum_{i=1}^S \phi_i = 1$. We use $\mathfrak{L}_i^{<m}$ to denote the set of products in $\mathfrak{L}_i$ that are preferred to product m. The choice probability ρ_n^m is given by

- $$\rho_n^m \quad := \quad \delta(1 - W^m(p_n^m)) \sum_{i=1}^{S} \phi_i \mathbb{I}_{m \in \mathfrak{L}_i} \prod_{j \in \mathfrak{L}_i^{<m}} W^j(p_n^j) \tag{1.4}$$

and $\rho_n^0 = 1 - \sum_m \rho_n^m$. Here $\mathbb{I}_{\mathcal{A}}$ represents the indicator function which is 1 when $\mathcal{A}$ holds, and 0 otherwise. Although we briefly use customer index as subscript of ϕ above to explain probability ϕ, the subscript of ϕ in the remainder is always a consideration set index or a product index when $M = 2$. Setting $\phi = \phi_1 = 1 - \phi_2$ is also a convention adopted for $M = 2$ in the remainder.

The choice probabilities in (1.2-1.3) are obtained with $(M, L) = (2, 2)$ and $\mathfrak{L}_1 = \{1, 2\}$, $\mathfrak{L}_2 = \{2, 1\}$. We also illustrate an example with $(M, L) = (3, 2)$, i.e., a population of customers choose among 3 products and each customer's consideration set has size 2. All possible considerations sets are $\mathfrak{L}_1 = \{1, 2\}$, $\mathfrak{L}_2 = \{1, 3\}$, $\mathfrak{L}_3 = \{2, 1\}$, $\mathfrak{L}_4 = \{3, 1\}$, $\mathfrak{L}_5 = \{2, 3\}$, $\mathfrak{L}_6 = \{3, 2\}$. Therefore from (1.4), $\rho_n^1 = \delta(1 - W^1(p_n^1))\{\phi_1 + \phi_2 + \phi_3 W^2(p_n^2) + \phi_4 W^3(p_n^3)\}$.

When all customers are offered the same price $p^{\mathcal{M}} := \{p^1, p^2, \ldots, p^M\}$, the choice probabilities in (1.1), (1.2-1.3) or (1.4) do not depend on the customer index n. Logit and WTP-choice models take the same price and sales data and output choice probabilities. Hence, they can be used in the same context and their comparison in the following three aspects is important.

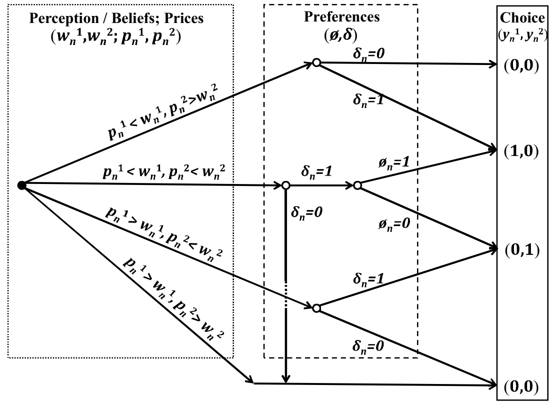


Figure 1.3. Alternative decision tree when prices are learnt first.

Sequences: A natural question is whether the sequence of events (learning prices and forming preferences) affect the outcome of the choice process. WTP-choice process above is conceived by assuming that customers learn prices in the last stage after they assess their preferences as in Figure 1.2(a). Figure 1.3 on the contrary shows customers that learn prices first as in an e-commerce context. We can check that the same events lead to the same choices in both Figures 1.2(a) and 1.3. Hence, WTP-choice model is robust with respect to the sequence of forming preferences and learning prices.

Logit model is not based on any explicit product sequence, as it assumes that a customer decides after collecting prices and assessing utilities for all products. Given choice probabilities, the probability for each sequence of considering products can be induced (Luce 1977). Although this gives a probability for such a sequence, it always requires consideration of all the products as the customers decide to purchase *at-once* after reaching the end of the sequence. Therefore, we can say that Logit model assumes at-once decision making. This is a fundamental difference between Logit and WTP-choice models.

Deciding at-once in Logit model can be argued to be a weakness that has led to nested Logit models (Danaher and Dagger 2012). In the nested models, there is a natural product hierarchy (sequence) and the customer chooses first a (product) group at the top level and then a product at the bottom level. To illustrate, we can consider the choice of a dessert (ice-cream, frozen yogurt, cakes) as the product group and then the choice of flavor (mint ice-cream, strawberry yogurt, chocolate cake) as the product. The sequence in the nested models is between levels not within each level. Sequencing within a level (such as ice-cream flavors) is an important feature of WTP-choice model and separates it from Logit models.

Information Requirement: Logit model requires customers that are informed with prices $p^{\mathcal{M}}$ to decide at-once, whereas WTP-choice model envisions customers that sequentially acquire price information only for the product under consideration. WTP-choice model on average requires less price information than Logit model, so the former model is more appropriate when the customers' search cost is high relative to the price (Cachon et al. 2005). For example, customer n can traverse the top path in Figure 1.2(a) to end up with the choice $(1, 0)$ without requiring p^2_n or w^2_n.

To avoid choosing an unavailable product, customers like to consider a product that is available in the retailer's inventory. This is incorporated into Logit model by dynamically dropping the stocked-out products from the consideration set (Mahajan and van Ryzin 2001). However, dynamically finding out stocked-out products in a market is more difficult than finding out prices. To overcome this difficulty, Hopp and Xu (2008) base Logit model on fill rates rather than stockouts as fill rates are more stable over time than stockouts. Similarly, we also provide an extension based on fill rates to incorporate stockouts in section 1.4. In this extension of WTP-Choice model, customers sequentially consider products so they need only information about the product currently under consideration. In sum, WTP-choice models require less price and inventory information than Logit models.

Independence of Irrelevant Alternatives (IIA) Property: Logit model has the IIA property, i.e., the relative odds of choice between two alternatives is not affected by the addition of another alternative. IIA property of Logit model has been criticized in the literature (Luce 1977, Train 2009). Mixed or nested version of Logit model or the exponomial choice model (Alptekinoğlu and Semple 2013) does not have the IIA property, nor does WTP-choice model. First, we calculate the relative odds in WTP-choice model with two options:

buy product 1 and *not buy*. This ratio is $\rho^1(p^1)/\rho^0(p^1) = \delta(1 - W^1(p^1))/\{\delta W^1(p^1) + 1 - \delta\}$. Adding another alternative – product 2 sold at price p^2 and its WTP distribution W^2 – we get another ratio $\rho^1(p^1, p^2)/\rho^0(p^1, p^2) = \delta(1 - W^1(p^1))\{(1 - \phi)W^2(p^2) + \phi\}/\{\delta W^1(p^1)W^2(p^2) + 1-\delta\}$. These ratios are not always identical, so addition of an alternative changes the relative odds.

1.3. Application of WTP-Choice Model to Competitive Pricing

Retailers often require a choice model that acts as an input for maximizing their profit. We consider a context where each retailer owns a product and determines its price. In particular, a retailer decides the price p^i of product i in expectation of the prices $p^{\mathcal{M}\setminus i} := p^{\mathcal{M}} \setminus p^i$ of the competing products. Equilibrium prices are studied under the following five settings: independent WTPs, dependent and continuous-valued WTPs, dependent and discrete-valued WTPs, price-dependent preference, and batch pricing for heterogeneous customers. The last four settings yield interesting insights, but the first setting of independent WTPs goes a long way to predict the sales in section 1.5.

1.3.1 Independent Willingness-to-pays

The objective for pricing product i to maximize the profit from a single customer is

$$\Pi^i(p^{\mathcal{M}}) = (p^i - c_i)\ \rho^i(p^1, p^2, \ldots, p^M) = (p^i - c_i)(1 - W^i(p^i))\Big[\delta \sum_{j=1}^{S} \phi_j \mathbb{I}_{i \in \mathfrak{L}_j} \prod_{k \in \mathfrak{L}_j^{<i}} W^k(p^k)\Big] \quad (1.5)$$

, where c_i is the cost per unit for retailer i. Since the terms above in the square brackets are constant in p^i, the response price $p^i(p^{\mathcal{M}\setminus i})$, i.e., the optimal price for given $p^{\mathcal{M}\setminus i}$, of retailer i is the maximizer of $(p^i - c_i)(1 - W^m(p^m))$. In other words, the response price $p^i(p^{\mathcal{M}\setminus i})$ to competitor prices $p^{\mathcal{M}\setminus i}$ is independent of $p^{\mathcal{M}\setminus i}$ when the WTPs are independent. Hence, the price p^i can be optimized without knowing competitor prices, the WTP distributions or preferences for the competitor products, and this makes the implementation of WTP-choice model easy. The response price p^i satisfies

$$\frac{p^i}{p^i - c_i} = p^i \left[\frac{dW^i(p^i)/dp^i}{1 - W^i(p^i)} \right] =: \Lambda^i(p^i), \tag{1.6}$$

where the term inside square brackets is the failure rate function of the WTP distribution $W^i(p^i)$ and $\Lambda^i(p^i)$ is the generalized failure rate function of the same distribution. Failure rate functions are well studied for many distributions, e.g., uniform, gamma, Weibull (with shape parameter > 1), truncated normal and modified extreme value distributions have increasing generalized failure rates, so the response price is unique. Besides providing such a uniqueness property, the optimality equation $\Lambda^i(p^i) = p^i/(p^i - c_i)$ is simple to solve, especially because $\Lambda^i(p^i)$ can be looked up from literature (pp.433-446 of Birolini 2010).

We assume that WTPs have increasing generalized failure rate unless otherwise is said. We suppose that they are distributed over intervals $[a_i, b_i)$ for $a_i, b_i \geq 0$ and allow for $a_i = 0$ and $b_i \to \infty$. If $a_i < c_i$, we can ignore WTPs lower than c_i and consider the rest, whose distribution is $W^i(\cdot)/(1 - W^i(c_i))$. Hence, we can assume $a_i \geq c_i$ without loss of generality. Then $W^i(a_i) = 0$ and $W^i(b_i) = 1$, so a customer buys when $p^i \leq a_i$ and nobody buys when $p^i \geq b_i$. The profit $\Pi^i(p^{\mathcal{M}})$ is strictly increasing in p^i as $\rho_i(p^{\mathcal{M}})$ is constant for $p^i \leq a_i$ and hence there cannot be a root for $\Lambda^i(p^i) = p^i/(p^i - c_i)$ with $p^i < a_i$. The objective is constant at zero for $p^i \geq b_i$, so it is not maximized by $p^i > b_i$. Therefore, when we refer to a unique root of $\Lambda^i(p^i) = p^i/(p^i - c_i)$, we effectively mean a root in the interval $[a_i, b_i)$. Theorem 1 establishes that this root maximizes the profit, and it summarizes the discussion above.

Theorem 1. (Loose coupling). *a) Unlike the profit function, the price response function of a retailer is independent of other retailer prices, so equilibrium prices are monopolistic prices.*

b) If $dW^i(p)/dp|_{p=a_i} = 0$ and $b_i \to \infty$, then $\Lambda^i(p^i) = p^i/(p^i - c_i)$ has a unique root.

c) If $\Lambda^i(p^i) = p^i/(p^i - c_i)$ has a unique root, the profit Π^i is unimodal in p^i.

As examples for WTP distribution $W^i(p)$, we consider uniform ($\square$) and shifted exponential ($\llcorner$). $W^i(p)$, $\Lambda^i(p)$ and the equilibrium prices p^{ie} are given in Table 1.1. Note that $\square$ and $\llcorner$ distributions have increasing generalized failure rates, consequently we have a unique best response price.

Table 1.1. Prices with different WTP distributions.

Distribution	$W^i(p)$	$\Lambda^i(p)$	p^{ie}
Uniform $\square$ (a_i, b_i)	$(p - a_i)/(b_i - a_i)$	$p/(b_i - p)$	$\max\{a_i, (b_i + c_i)/2\}$
Shifted exponential $\llcorner$ $(a_i, \infty; \tau_i)$	$1 - \exp(-\tau_i(p - a_i))$	$p\tau_i$	$c_i + 1/\tau_i$

For Logit model, there is not a simple expression for the response price. The objective $(p^i - c_i)\lambda^i(p^{\mathcal{M}})$ yields an implicit equation, where both sides depend on p^i and the right-hand side depends on also $p^{\mathcal{M}\setminus i}$:

$$p^i = c_i - [\beta\{1 - \lambda^i(p^i, p^{\mathcal{M}\setminus i})\}]^{-1}. \tag{1.7}$$

Comparison of (1.6) and (1.7) shows the simplicity of pricing with WTP-choice model. Standard logit model applies only to the case of independent utilities without inventory consideration, so our next comparison of Logit and WTP-choice model takes place in section 1.5.

Loose coupling in Theorem 1 is a striking result that relates to the monopolistic competition. Monopolistic competition occurs when prices change slightly and customers resist to switching from one product to another. This resistance is captured more by price-independent preferences than their dependent counterparts. When the preferences are dependent on prices, one may expect loose coupling to fail, which we show in section 1.3.4. However, it is not clear without a rigorous analysis if the loose coupling property holds in the cases of dependent WTPs or batch pricing for heterogenous customers. We analyze these in a duopoly – a market with two firms, each of which sells a product. For example, Fedex with UPS and AutoZone with O'Reilly Automotive constitute a duopoly, respectively, in *Air Freight* and in *Automotive Retail*. A firm in a market with multiple firms usually benchmarks itself against another firm that often leads in terms of revenue. For example, Target is a *General Merchandise Store*, a category led by Wal-Mart in revenue. So Target can be paired with Wal-Mart to have a duopoly, despite the presence of smaller competing firms.

1.3.2 Dependent and Continuous-Valued Willingness-to-pays

To extend WTP-choice model to dependent WTPs, we start with the objective of pricing product i

$$\Pi^i(p^i, p^{-i}) = (p^i - c_i)\{\phi_i \mathrm{P}(W^i \geq p^i) + \phi_{-i}\mathrm{P}(W^i \geq p^i, W^{-i} \leq p^{-i})\}, \tag{1.8}$$

where index $-i$ denotes the retailer other than retailer i. When $i = 1$ for example, ϕ_i and ϕ_{-i} are the probabilities that a customer respectively prefers product 1 and 2. We consider two examples: identical WTPs and identically distributed WTPs. We determine the equilibrium prices to identify if these prices inherit dependence (coupling) from WTPs.

Identical WTPs: Products which are ideal substitutes or very similar can have identical WTPs. For such WTPs, we assume uniform distribution so $W^1 = W^2 \sim W = \square[a, b]$. Therefore, $\mathrm{P}(W^i \leq p) = (p - a)/(b - a)$ if $a \leq p \leq b$; 0 otherwise. $\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = \mathrm{P}(p^1 \leq W \leq p^2) = (p^2 - p^1)/(b - a)$ if $p^2 > p^1$; 0 otherwise.

When retailers set identical prices, i.e., $p^1 = p^2$, $\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = 0$, the equilibrium prices are monopolistic prices, i.e., $p^i = (b + c_i)/2$. This is a symmetric equilibrium with identical prices $(p^{1e}, p^{2e}) = ((b + c)/2, (b + c)/2)$ when retailers have identical costs $c_1 = c_2 = c$.

When retailers charge different prices, say $p^1 < p^2$, we have structurally different profit maximization problems for retailers 1 and 2.

$$\text{Retailer 1:} \quad \max_p (p - c_1)\{\phi(b - p)/(b - a) + (1 - \phi)(p^2 - p)/(b - a)\} \quad \text{if } p < p^2,$$

$$\text{Retailer 2:} \quad \max_p (p - c_2)\{(1 - \phi)(b - p)/(b - a)\} \quad \text{if } p > p^1.$$

The best response price for a retailer depends on the price of the other retailer, so loose coupling does not hold; $p^1(p^2) = (\phi b + (1-\phi)p^2 + c_1)/2$ and $p^2(p^1) = (c_2+b)/2$. The equilibrium is $(p^{1e}, p^{2e}) = (((1+\phi)b + (1-\phi)c_2 + 2c_1)/4, (c_2+b)/2)$ if $p^{1e} < p^{2e}$. Similarly for $p^{1e} > p^{2e}$, (p^{1e}, p^{2e})= $((c_1+b)/2, ((2-\phi)b + \phi c_1 + 2c_2)/4)$. Therefore, when $2c_1 - (1-\phi)b < (1+\phi)c_2$ we have $p^{1e} < p^{2e}$; when $(2-\phi)c_1 + \phi b > 2c_2$ we have $p^{1e} > p^{2e}$. As these conditions are not mutually exclusive, it is possible to have both equilibria. When retailers are identical, i.e., $c_1 = c_2 = c, \phi = 1-\phi = 0.5$, both conditions are satisfied on account of $c < b$. Then we have two non-symmetric equilibria. This is very interesting as retailers charge different equilibrium prices $(p^{1e}, p^{2e}) = ((3b+5c)/8, (c+b)/2)$ and $(p^{1e}, p^{2e}) = ((c+b)/2, (3b+5c)/8)$, even if they are identical. The prices at these equilibria are mirror images of each other with respect to the $p^1 = p^2$ line.

Identically distributed WTPs: We consider $W \sim \square[0,b]$, $W^1 \sim W + \square[0,\epsilon]$ and $W^2 \sim W + \square[0,\epsilon]$ for $0 < \epsilon < b - c$. The same W is a part of both W_1 and W_2, while realizations of $\square[0,\epsilon]$ can be different.

Lemma 1. (Equilibrium with Identically distributed WTPs). *a) For $\epsilon \le p^1, p^2 \le b$, we have $\mathrm{P}(W^i \ge p^i) = (\epsilon + 2(b-p^i))/(2b)$ and*

$$\mathrm{P}(W^1 \ge p^1, W^2 \le p^2) = \begin{cases} 0 & \text{if } p^2 \le p^1 - \epsilon, \\ (p^2 - p^1 + \epsilon)^3/(6b\epsilon^2) & \text{if } p^1 - \epsilon \le p^2 \le p^1, \\ ((p^1-p^2)^3 + 3(p^1-p^2)^2\epsilon - 3(p^1-p^2)\epsilon^2 + \epsilon^3)/(6b\epsilon^2) & \text{if } p^2 - \epsilon \le p^1 \le p^2, \\ (p^2 - p^1)/b & \text{if } p^1 \le p^2 - \epsilon. \end{cases}$$

b) For identical retailers, the only symmetric equilibrium has $p^{1e} = p^{2e} = (6b + 9c + 4\epsilon)/15$. There are also two non-symmetric equilibria as $(p^{1e}, p^{2e}) = ((8b + 8c + 4\epsilon)/16, (6b + 10c + 3\epsilon)/16)$ satisfying $p^{1e} \ge p^{2e} + \epsilon$ and $(p^{1e}, p^{2e}) = ((6b + 10c + 3\epsilon)/16, (8b + 8c + 4\epsilon)/16)$ satisfying $p^{1e} \le p^{2e} - \epsilon$.

The symmetric equilibrium points to an equal market split. In non-symmetric equilibria, the retailer charging more has a smaller market share. Although both retailers are identical, the market can have a retailer leading with a higher price and the other leading with a larger market share.

When the WTPs are dependent as opposed to independent, we obtain lower equilibrium prices. Using $\mathrm{P}(W^i \geq p^i) = (\epsilon + 2(b - p^i))/(2b)$ and loose coupling, the equilibrium price is $p^{ie} = (2b + 2c + \epsilon)/4$ for uniformly distributed independent WTPs and identical retailers. When the WTPs are dependent, the symmetric equilibrium price $p^{1e} = p^{2e} = (6b+9c+4\epsilon)/15$ is lower than $(2b + 2c + \epsilon)/4$. Similarly the non-symmetric equilibria satisfy $(p^{1e} + p^{2e})/2 < (2b+2c+\epsilon)/4$. Recognition of dependence in our example reduces prices, which is a welcome news to customers but not so to firms. Ideally, firms should reduce the dependence of WTPs, possibly by employing product differentiation strategies.

1.3.3 Dependent and Discrete-Valued Willingness-to-pays Lead to Price Cycles

A price cycle is a dynamic price equilibrium identified by a finite sequence of non-identical multiple price-pairs, which satisfies three conditions: i) any consecutive pair must share a *common price*; ii) the *uncommon price* in the succeeding pair is the best response to the *common price*; iii) when the last and the first price pairs in the sequence are considered as consecutive price pairs, their prices satisfy conditions i) and ii). We consider discrete-valued WTPs in this section because they can cause a price cycle. Discrete WTPs imply discrete optimal prices – often found in practice as multiples of ¢1 or ¢5 (Phillips 2005).

In the absence of a price-pair equilibrium, it is possible to seek a price cycle that consists of multiple price-pairs. When there is not a price-pair equilibrium, we can start at an arbitrary price-pair and generate a price sequence that satisfies i) and ii). Since price-pairs are finite, such a sequence must also satisfy iii). Hence, absence of a price-pair equilibrium implies the presence of at least one price cycle. The contrapositive of this statement is also true; absence of a price cycle implies the presence of a price-pair equilibrium. Moreover, price cycles and price-pair equilibria may co-exist in a given instance. Consequently, a price cycle can be the best description of the equilibrium in a market that does not have a price-pair equilibrium.

A price cycle can be represented by a sequence of price-pairs: $\{p_0^1, p_0^2\} \rightarrow \{p_0^1, p_1^2\} \rightarrow \{p_1^1, p_1^2\} \rightarrow \{p_1^1, p_2^2\} \rightarrow \cdots \rightarrow \{p_j^1, p_0^2\} \rightarrow \{p_0^1, p_0^2\}$, where '$\rightarrow$' indicates the direction of a price cycle. For example, $\{p_1^1, p_1^2\} \rightarrow \{p_1^1, p_2^2\}$ implies that the cycle goes from $\{p_1^1, p_1^2\}$ to $\{p_1^1, p_2^2\}$, as p_2^2 is the best response of the retailer 2 to retailer 1's price p_1^1. The length of the price cycle is the minimum number of price-pairs traversed before returning to the same price-pair. Accordingly, the shortest price cycle is of length 4.

An example of a price cycle with length 4 is depicted by arrows in Table 1.2, which shows the joint WTP probabilities for prices $p^1 \in \{1, 2, 7\}$ and $p^2 \in \{1, 2, 3\}$, respectively, charged by retailers 1 and 2. In the example, retailers incur zero cost and $\phi = 0.4$. The expected profits of the retailers are from (1.8), e.g., $\Pi^1(2,2) = 2\,(\phi \mathrm{P}(W^1 \geq 2) + (1-\phi)\mathrm{P}(W^1 \geq 2, W^2 < 2))$ $= 2\ (0.4\ (0.7) + 0.6\ (0.3)) = 0.92$, other profits are in Table 1.2. They satisfy $\Pi^2(2,3) > \Pi^2(2,1), \Pi^2(2,2)$; $\Pi^1(7,3) > \Pi^1(1,3), \Pi^1(2,3)$; $\Pi^2(7,2) > \Pi^2(7,1), \Pi^2(7,3)$ and $\Pi^1(2,2) > \Pi^1(1,2), \Pi^1(7,2)$, and these four inequalities respectively justify the 4 arrows in the cycle $\{2,2\} \rightarrow \{2,3\} \rightarrow \{7,3\} \rightarrow \{7,2\} \rightarrow \{2,2\}$. This is the unique cycle and there does not exist a single price-pair equilibrium.

Table 1.2. Price cycle example. WTP probabilities; Profits $(\Pi^1(p^1, p^2), \Pi^2(p^1, p^2))$.

		Retailer 2		
	(p^1, p^2)	1	2	3
	1	0.00; (0.40, 0.60)	0.05; (0.58, 0.84)	0.25; (0.73, 0.81)
Retailer 1	2	0.25; (0.56, 0.72)	0.10; (0.92, 1.08) $\rightarrow$	0.10; (1.16, 1.11) $\downarrow$
	7	0.05; (0.70, 0.90)	0.10; (0.91, 1.24) $\uparrow$	0.10; (1.33, 1.23) $\leftarrow$

The dependence of WTPs can eliminate a price-pair equilibrium and leads to a price cycle as in the above example. If they are independent, there is always a price-pair equilibrium by Theorem 1. On the other hand, even if there is a price-pair equilibrium, WTPs can be dependent. This is because WTP dependence can be induced by altering WTP probabilities that do not show up in the equilibrium comparisons. So inferring independence is harder, but still possible as in parts a) and b) of the next theorem. It is also important to characterize the absence of a price cycle towards concluding that a price-pair equilibrium exists under dependent WTPs, this reasoning is adopted by the theorem.

Theorem 2. (Equilibrium with dependent WTPs). *a) There is no price cycle of length 4 such as* $\{p_l^1, p_l^2\} \to \{p_l^1, p_h^2\} \to \{p_h^1, p_h^2\} \to \{p_h^1, p_l^2\} \to \{p_l^1, p_l^2\}$*, if the WTPs satisfy*

$$\mathrm{P}(W^i \geq p_l^i,\ W^{-i} < p_h^{-i})\mathrm{P}(W^i \geq p_h^i,\ W^{-i} < p_l^{-i}) = \mathrm{P}(W^i \geq p_h^i,\ W^{-i} < p_h^{-i})\mathrm{P}(W^i \geq p_l^i,\ W^{-i} < p_l^{-i}).$$

b) The WTPs are independent if the condition in b) is satisfied for all prices.

c) No price cycle of length 4 can contain a price pair with the lowest prices for both retailers. There is a price-pair equilibrium if both retailers consider binary prices $p^i \in \{p_l^i, p_h^i\}$.

d) There is no price cycle of length 4 if WTPs and preference (ϕ_i) *satisfy*

$$\max_{\{p_l^i, p_h^i, p^{-i}\}} \left\{ \frac{P(p_h^i \leq W^i, p^{-i} > W^{-i}) + \phi_i P(p_h^i \leq W^i, p^{-i} \leq W^{-i})}{P(p_l^i \leq W^i < p_h^i, p^{-i} > W^{-i}) + \phi_i P(p_l^i \leq W^i < p_h^i, p^{-i} \leq W^{-i})} \right\} \leq \frac{a_i - c_i}{b_i - a_i}$$

for either $i = 1$ *or* 2.

e) There is a price-pair equilibrium if either retailer considers binary prices $p^i \in \{p_l^i, p_h^i\}$ *and the condition in d) is satisfied.*

Theorem 2.a) gives a condition to eliminate a particular cycle. This condition boils down to independence of WTPs when all cycles of length 4 are to be ruled out. So independence is sufficient to eliminate these cycles. Theorem 2.a-b) eliminate the cycles whereas Theorem 1 establishes the existence of a price-pair equilibrium, which does not rule out cycles. Theorem 2.c) shows that when retailers consider binary prices, there must be a price-pair equilibrium despite the dependence of the WTPs. Theorem 2.d) gives the condition under which there is no price cycle of length 4 despite the dependence of WTPs. From the WTP distribution in Table 1.2, we evaluate the left-hand side of the condition in Theorem 2.d) for $i = 1$ and $i = 2$. Correspondingly, if either $\frac{39}{15} \leq \frac{a_1}{b_1 - a_1}$ or $\frac{37}{17} \leq \frac{a_2}{b_2 - a_2}$ holds, there are no price cycles of length 4. These inequalities imply conditions on the support parameters $\frac{a_1}{b_1} \geq \frac{39}{54}$ or $\frac{a_2}{b_2} \geq \frac{37}{54}$. So if the support of either W^1 or W^2 is tight, i.e., the uncertainty of W^1 or W^2 is low, there are no price cycles of length 4. This conclusion leads to price-pair equilibrium in Theorem 2.e) when either retailer considers binary prices. In these regards, Theorem 2 applies even when Theorem 1 does not.

1.3.4 Price-Dependent Preference

Another extension of WTP-choice model involves price dependent preference $\phi(p^1, p^2)$ that can be used to capture some of the real-life contexts, where not only profits but also price responses are coupled. In a duopoly profit maximization problem with continuous WTPs, retailer 1's objective is $\max_p (p - c_1)(1 - W^1(p))\{\phi(p, p^2) + (1 - \phi(p, p^2))W^2(p^2)\}$. The best response price p^1 for retailer 1 satisfies:

$$1 = \frac{c_1}{p^1} + \left[\Lambda^1(p^1) - \frac{p^1(1 - W^2(p^2))[\partial\phi(p^1, p^2)/\partial p^1]}{\{\phi(p^1, p^2) + (1 - \phi(p^1, p^2))W^2(p^2)\}}\right]^{-1}. \tag{1.9}$$

It is easy to see from above that the best response p^1 for retailer 1 depends on other retailer's price p^2, unlike loose coupling in Theorem 1. This coupling can lead to higher or lower price responses and is shown with two examples in Table 1.3. In particular, a retailer responds by charging a higher (lower) price if the preference for that retailer is increasing (decreasing) in its own price compared to the price charged when the preference is independent of prices.

Theorem 3. (Equilibrium with price-dependent preferences). *If the preferences for both retailers are increasing (decreasing) in their prices and an equilibrium exists, then the equilibrium prices will be higher (lower) compared to the equilibrium prices when the preferences are independent of prices.*

Table 1.3. Examples with dependent symmetric preferences.

Symmetric Preference	Response Price
Increasing in p^1	p^1 is higher than the price under price-independent preference
$\phi(p^1, p^2) = \dfrac{p^1}{p^1 + p^2}$	$1 = \dfrac{c_1}{p^1} + \left[\Lambda^1(p^1) - \dfrac{p^1 p^2 (1 - W^2(p^2))}{(p^1 + p^2)\{p^1 + p^2 W^2(p^2)\}} \right]^{-1}$
Decreasing in p^1	p^1 is lower than the price under price-independent preference
$\phi(p^1, p^2) = \dfrac{p^1_{max} - p^1}{p^1_{max} + p^2_{max} - p^1 - p^2}$	$1 = \dfrac{c_1}{p^1} + \left[\Lambda^1(p^1) + \dfrac{p^1 p^2 (1 - W^2(p^2))}{\{p^1_{max} p^2_{max} W^2(p^2) - p^2 \left(p^1_{max} - p^1\right)\left(1 - W^2(p^2)\right)\}} \right]^{-1}$

1.3.5 Batch Pricing for Heterogeneous Customers

Retailers may resort to batch pricing for a population of customers, where each customer n has a different WTP distribution W_n^i and a generalized failure rate Λ_n^i when buying from retailer i. For two customers, retailer 1's objective is

$$\max_{p^1}(p^1 - c_1)\left\{(1 - W_1^1(p^1))\{(1-\phi)W_1^2(p^2)+\phi\} + (1 - W_2^1(p^1))\{(1-\phi)W_2^2(p^2)+\phi\}\right\}.$$

The first order condition for profit maximization gives us

$$\frac{p^1}{p^1 - c_1} = \Lambda_1^1(p^1)\frac{\rho_1^1(p^1,p^2)}{\rho_1^1(p^1,p^2)+\rho_2^1(p^1,p^2)} + \Lambda_2^1(p^1)\frac{\rho_2^1(p^1,p^2)}{\rho_1^1(p^1,p^2)+\rho_2^1(p^1,p^2)}. \quad (1.10)$$

Price response $p^1(p^2)$ then depends on ϕ, p^2 and W^2, unlike the cases of individual pricing or homogenous customers. That is, loose coupling disappears because of batch pricing.

1.4. WTP-choice Models and Prices under Stockouts

Consideration of stockouts (inventory unavailability) can improve the applicability of a choice model. A stockout at a retailer increases the demand at another retailer as customers substitute their preferred but stocked-out product with another. We can investigate the versatility of WTP-choice model and test the robustness of loose coupling under stockouts. We consider independent WTPs because the loose coupling property fails already with dependent WTPs, the solution with independent WTPs can approximate the solution with dependent WTPs, and the WTP-Choice model with independent WTPs represent real-life data sufficiently well as discussed in section 1.5.

We use fill rates to study competitive pricing under stockouts in a duopoly. If a customer arrives at a stocked-out retailer, he naturally considers buying from another retailer; this can be called inventory-based substitution behavior. Otherwise, if stockout-facing customers do not consider buying from another retailer and simply buy nothing, the expected demand faced by a retailer is simply his fill rate times the demand without stockouts. Subsequently, competitive pricing follows the same structure (including loose coupling) as before and is not interesting to analyze.

Under inventory-based substitution, customers preferring retailer 2, in particular, consider retailer 1 when retailer 2 is stocked-out. Previously, such customers considered retailer 1 only when retailer 2 prices too high. Now both a stockout and a high price at retailer 2 divert customers to retailer 1, hence the choice probability of retailer 1 needs to have some additional terms based on stockouts. Assuming $\delta = 1$ in the remainder, these additional terms are detailed below depending on whether customers tolerate backorders or not and whether they give priority to immediate availability or to preferred retailer when they backorder. Throughout these cases, the stockout probability or *stockout rate* at retailer i is denoted by ν_i , i.e., $1 - \nu_i$ is the fill rate of retailer i.

1.4.1 Lost Sales

A customer sooner or later, depending on preferring retailer i or the other, shows up at retailer i with the probability $\phi_i + \phi_{-i}[(1-\nu_{-i})W^{-i}(p^{-i}) + \nu_{-i}]$. This customer finds the product in stock with fill rate $1-\nu_i$ and buys with probability $1 - W^i(p^i)$. Hence, the sales probability is

$$\rho^i_{ls}(p^1, p^2) \;\; := \;\; (1-\nu_i)(1-W^i(p^i))[\phi_i + \phi_{-i}((1-\nu_{-i})W^{-i}(p^{-i}) + \nu_{-i})]. \qquad (1.11)$$

When stockouts are considered, we use the term sales probability rather than choice probability. The difference between a choice and a sale is the inventory availability incorporated via ν_i. If both retailers are stocked-out or price too high for a particular customer, they lose the customer. This happens with probability $[\nu_i + (1-\nu_i)W^i(p^i)][\nu_{-i} + (1-\nu_{-i})W^{-i}(p^{-i})]$. Since $\nu_i + (1-\nu_i)W^i(p^i) \geq W^i(p^i)$, the loss probability is at least $\rho^0(p^1, p^2) = W^1(p^1)W^2(p^2)$ of section 1.2 for $\delta = 1$. Eventually,

$$\text{Profit of retailer } i \text{ under lost sales} := (p^i - c_i)\rho^i_{ls}(p^1, p^2).$$

More details on obtaining profit expressions with a specific inventory policy are in Appendix A.

Equipped with profit expressions, we can express the response price for retailer i:

$$p^i_{ls}(p^{-i}) = \arg\max_{p^i} \left\{ \Pi^i_{ls}(p^1, p^2) = (p^i - c_i)(1 - W^i(p^i))(1-\nu_i)[\phi_i + \phi_{-i}((1-\nu_{-i})W^{-i}(p^{-i}) + \nu_{-i})] \right\}. \tag{1.12}$$

The lost sales profit Π^i_{ls} has multiplication of three terms, the first two are $(p^i - c_i)$ and $1 - W^i(p^i)$, and they depend on retailer i's price p^i while the last term in brackets is not dependent on p. So, this profit expression has the same structure as (1.5). Hence, the price solution p^i_{ls} for the case of lost sales is the same as that without stockouts as in (1.6).

$$p^i_{ls} - c_i \quad = \quad \frac{1 - W^i(p^i_{ls})}{dW^i(p^i_{ls})/dp^i_{ls}}. \tag{1.13}$$

The prices of Table 1.1 as well as loose coupling still remain valid. More interestingly, the equilibrium price charged by a retailer does not depend on the stockout rates, as the price is relevant only when the retailer can fulfill the demand. However, the profit Π^i_{ls} depends heavily on both ν_i and ν_{-i}.

1.4.2 Backorders

To study pricing under backorders, we first present two types of customer behavior in the event of a stockout. These behaviors stem from the customer priority attached to the *preferred retailer* versus the *immediate availability* of the product. Facing a stockout at his preferred retailer, would a customer backorder from his preferred retailer or visit a non-preferred retailer? Backordering customers favor their preferred *retailer*. Other customers, visiting another retailer, favor the prospect of immediate *availability*. Associated with retailer favoring customers, we present the decision tree in Figure 1.4, and the figure showing the decision tree of availability favoring customers is in Appendix A.

In Figure 1.4, a backorder with retailer i is denoted by y^i_b. Appending this to the choice vector, we obtain the sales vector $(y^1, y^1_b; y^2, y^2_b)$. The sales probabilities are

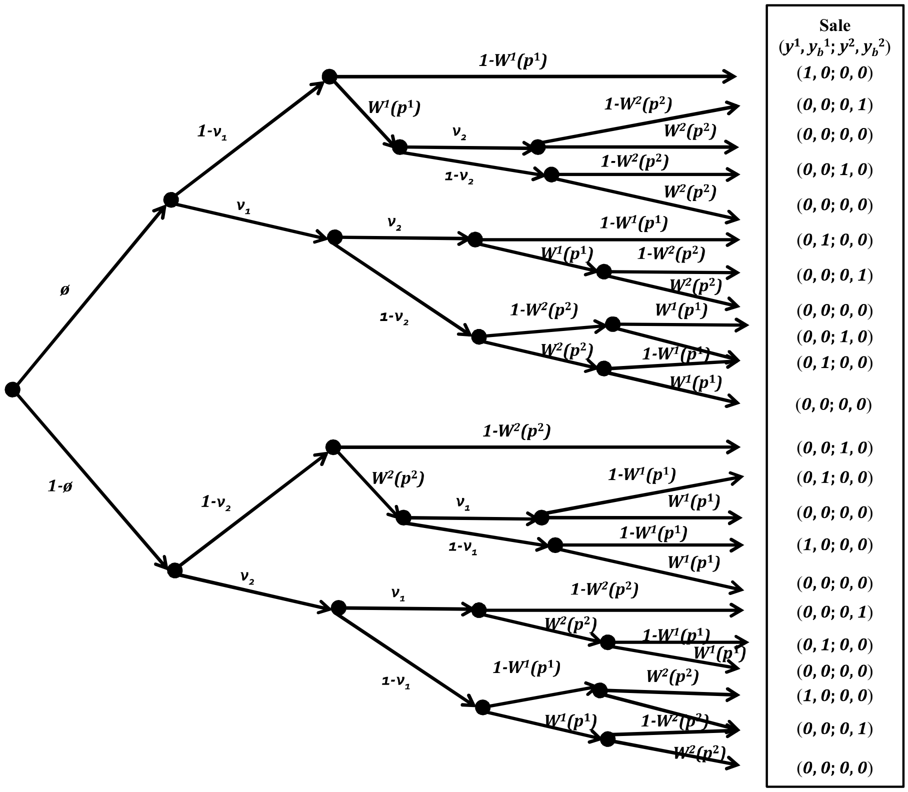


Figure 1.4. A retailer favoring customer's decision tree from the firm's perspective.

- $\rho^{1,n} \;:=\; \mathrm{P}\left((y^1, y_b^1; y^2, y_b^2) = (1,0;0,0)\right),$
- $\rho^{2,n} \;:=\; \mathrm{P}\left((y^1, y_b^1; y^2, y_b^2) = (0,0;1,0)\right),$
- $\rho^{1,b} \;:=\; \mathrm{P}\left((y^1, y_b^1; y^2, y_b^2) = (0,1;0,0)\right),$
- $\rho^{2,b} \;:=\; \mathrm{P}\left((y^1, y_b^1; y^2, y_b^2) = (0,0;0,1)\right),$

where superscripts "n" and "b" respectively indicate selling now and backordering. To indicate retailer and availability favoring behaviors subscripts ret and ava are used in sales probabilities, respectively.

For retailer favoring customers, we can obtain sales probabilities from Figure 1.4.

$$\begin{aligned}
\rho_{ret}^{i,n}(p^1,p^2) &= (1-\nu_i)(1-W^i(p^i))\left[\phi_i + \phi_{-i}\{(1-\nu_{-i})W^{-i}(p^{-i}) + \nu_{-i}W^{-i}(p^{-i})\}\right] \\
&= (1-\nu_i)(1-W^i(p^i))\left[\phi_i + \phi_{-i}W^{-i}(p^{-i})\right], \qquad (1.14) \\
\rho_{ret}^{i,b}(p^1,p^2) &= \nu_i(1-W^i(p^i))\left[\phi_i + \phi_{-i}W^{-i}(p^{-i})\right], \qquad (1.15)
\end{aligned}$$

The probability of buying now or backordering from retailer i is $\rho_{ret}^{i,n}(p^1,p^2) + \rho_{ret}^{i,b}(p^1,p^2) = (1-W^i(p^i))[\phi_i + \phi_{-i}W^{-i}(p^{-i})]$, which is equal to $\rho^i(p^1,p^2)$ in (1.2) for $\delta = 1$. That is, the sales probability $\rho^i(p^1,p^2)$ is split by proportions $1-\nu_i$ and ν_i respectively into probabilities of sales now and of backorder. An arbitrary customer ends up at retailer i with probability $[\phi_i + \phi_{-i}W^{-i}(p^{-i})]$ and buys with probability $1-\nu_i$. The probability of not buying from either retailer is $1-(\rho^{1,n}_{ret}(p^1,p^2)+\rho^{1,b}_{ret}(p^1,p^2))-(\rho^{2,n}_{ret}(p^1,p^2)+\rho^{1,b}_{ret}(p^1,p^2))$, which coincides with $\rho^0(p^1,p^2)=W^1(p^1)W^2(p^2)$ of section 1.2.

For availability favoring customers,

$$\rho^{i,n}_{ava}(p^1,p^2) \;=\; (1-\nu_i)(1-W^i(p^i))\left[\phi_i+\phi_{-i}\{(1-\nu_{-i})W^{-i}(p^{-i})+\nu_{-i}\}\right], \quad (1.16)$$

$$\rho^{i,b}_{ava}(p^1,p^2) \;=\; \nu_i(1-W^i(p^i))\left[\phi_i\{\nu_{-i}+(1-\nu_{-i})W^{-i}(p^{-i})\}+\phi_{-i}W^{-i}(p^{-i})\right], \quad (1.17)$$

Probabilities $\rho^{1,n}_{ava}(p^1,p^2)$, $\rho^{1,b}_{ava}(p^1,p^2)$, $\rho^{2,n}_{ava}(p^1,p^2)$ and $\rho^{2,b}_{ava}(p^1,p^2)$ also sum up to $1-\rho^0(p^1,p^2)$, where $\rho^0(p^1,p^2)=W^1(p^1)W^2(p^2)$ is the probability of not buying from either retailer.

The probabilities of buying and backordering are compared between the settings of retailer favoring and availability favoring customers, while the probability of not buying is $\rho^0(p^1,p^2)$ in both settings. Comparing sales probabilities in (1.14) and (1.16), we see the difference between availability favoring and retailer favoring customers. This difference between sales probabilities is $(1-\nu_i)(1-W^i(p^i))[\phi_{-i}\nu_{-i}(1-W^{-i}(p^{-i}))]$, where the part in square brackets represents the probability of an availability favoring customer to switch from preferred retailer $-i$ to retailer i when he is willing to pay at least the price p^{-i} to retailer $-i$ who is stocked-out. This probability shows up in (1.16) but not in (1.14), so $\rho^{i,n}_{ava}(p^1,p^2)\geq\rho^{i,n}_{ret}(p^1,p^2)$. That is, availability favoring behavior leads to a higher probability of sales now than retailer favoring behavior. Comparing (1.15) and (1.17), we see more differences. The difference between backorder probabilities is $\nu_i(1-W^i(p^i))[\phi_i(1-\nu_i)(1W^{-i}(p^{-i}))]$, where the part in square brackets represents the probability of a retailer favoring customer to stick with preferred retailer i rather than switching to retailer $-i$ that has inventory and reasonable price p^{-i}. Thus, we obtain $\rho^{i,b}_{ava}(p^1,p^2)\leq\rho^{i,b}_{ret}(p^1,p^2)$, i.e., availability favoring behavior leads to a lower probability of backorders than retailer favoring behavior.

In addition to WTP-choice model of no inventory consideration in section 1.2, this section presents three WTP-Choice models: lost sales, retailer and availability favoring customers. The sales probabilities for these are respectively presented in (1.2-1.3), (1.11), (1.14-1.15) and (1.16-1.17) and summarized in Table 1.4 after introducing the *probability of retaining* (a customer at his preferred retailer) and the *probability of overflow* (of a customer from his preferred retailer to the other) for ease of reference. Interestingly, both of these probabilities under all four models take only one of the four values 0, 1, $W^{-i}(p^{-i})$ or $W^{-i}(p^{-i}) + \nu_{-i}(1 - W^{-i}(p^{-i}))$. There are no backorders without inventory consideration or with lost sales, therefore corresponding rows in Table 1.4 are not shown. In the remainder, we focus on markets where either customers are all availability favoring or retailer favoring. For a more general setting in a different study, these two types of customers can be mixed with a certain probability.

Customers in a Monopolistic Market: To express profits, let the backorder cost be d_i for retailer i, which is a penalty cost charged for each stocked-out unit. It can relate to the additional cost of overtime, of using a more responsive source/transportation mode so that customers agree to backorder instead of walking away (p.65 of Porteus 2002). The retailer earns $p^i - c_i$ when a unit is sold from inventory and $p^i - c_i - d_i$ when a unit is backordered. Note that retailer favoring customers cannot be distinguished from the availability favoring customers in a monopoly. In both cases, we resort to the objective $\max_{p^i \geq 0}(p^i - c_i - \nu_i d_i)(1 - W^i(p^i))$, whose maximizer satisfies $p^i - c_i - \nu_i d_i = \frac{1-W^i(p^i)}{dW^i(p^i)/dp^i}$. Then the optimal prices under various distributions are exactly those in the last column of Table 1.1, except that $c_i + \nu_i d_i$ should replace c_i in that column.

Retailer Favoring Customers in a Duopolistic Market: When customers are retailer favoring and we have backorders, then the probability of sale is $\rho^{i,n}_{ret} + \rho^{i,b}_{ret}$ and the expected profit from this sale is $p^i - c_i - \nu_i d_i$. Accordingly, the expected profit of retailer i is

Profit of Retailer i from Retailer Favoring Customers: $(p^i - c_i - \nu_i d_i)(\rho^{i,n}_{ret}(p^1, p^2) + \rho^{i,b}_{ret}(p^1, p^2))$,

Table 1.4. Summary of probabilities in four WTP models.

	[Prob. of retaining] and [Prob. of overflow] satisfy	
[Prob. of Sale now] in $\{\rho^i, \rho^i_{ls}, \rho^{i,n}_{ret}, \rho^{i,n}_{ava}\}$	$= (1-\nu_i)(1-W^i(p^i))\{\phi_i$[Prob. of retaining] $+ \phi_{-i}$[Prob. of overflow]$\}$	
Models	Prob. of retaining	Prob. of overflow
No inventory consideration $\nu_i = 0$	1	$W^{-i}(p^{-i})$
Lost sales	1	$W^{-i}(p^{-i}) + \nu_{-i}(1 - W^{-i}(p^{-i}))$
Retailer favoring	1	$W^{-i}(p^{-i})$
Availability favoring	1	$W^{-i}(p^{-i}) + \nu_{-i}(1 - W^{-i}(p^{-i}))$
	[Prob. of retaining] and [Prob. of overflow] satisfy	
[Prob. of Backorder] in $\{\rho^{i,b}_{ret}, \rho^{i,b}_{ava}\}$	$= \nu_i(1-W^i(p^i))\{\phi_i$[Prob. of retaining] $+ \phi_{-i}$[Prob. of overflow]$\}$	
Models	Prob. of retaining	Prob. of overflow
Retailer favoring	1	$W^{-i}(p^{-i})$
Availability favoring	$W^{-i}(p^{-i}) + \nu_{-i}(1 - W^{-i}(p^{-i}))$	$W^{-i}(p^{-i})$

which is explained more in Appendix A. We can write this profit explicitly as

$$\Pi^i_{ret}(p^1, p^2) := (p^i - c_i - \nu_i d_i)[1 - W^i(p^i)][\phi_i + \phi_{-i} W^{-i}(p^{-i})]. \tag{1.18}$$

$\Pi^i_{ret}(p^1, p^2)$ has three terms, first two are $(p^i - (c_i + \nu_i d_i))\,[1 - W^i(p^i)]$ and they depend on the price p^i, while the third term is independent of p^i. Leaving the third term aside, the optimal price can be found by maximizing the first two terms and satisfies

$$p^i_{bo} - c_i - \nu_i d_i = \frac{1 - W^i(p^i_{bo})}{dW^i(p^i_{bo})/dp^i_{bo}}, \tag{1.19}$$

which is exactly the price charged by the monopolist. Therefore, loose coupling of Theorem 1 extends to a competitive market with retailer favoring customers.

Availability Favoring Customers in a Duopolistic Market: Following the discussion for retailer favoring customers, we have the profit of retailer i from availability favoring customers as:

$$(p^i - c_i - \nu_i d_i)(\rho^{i,n}_{ava}(p^1, p^2) + \rho^{i,b}_{ava}(p^1, p^2)).$$

The expected profit for retailer i can be written explicitly as

$$\begin{aligned}\Pi^i_{ava}(p^1, p^2) \;=\; & (p^i - c_i - \nu_i d_i)\left[1 - W^i(p^i)\right]\Big\{(1 - \nu_i)\left[\phi_i + \phi_{-i}\left\{(1 - \nu_{-i})W^{-i}(p^{-i}) + \nu_{-i}\right\}\right] \\ & + \nu_i\left[\phi_i\left\{\nu_{-i} + (1 - \nu_{-i})W^{-i}(p^{-i})\right\} + \phi_{-i}W^{-i}(p^{-i})\right]\Big\}. \end{aligned} \tag{1.20}$$

This profit is structurally identical to Π^i_{ret} in (1.18) as it differs only by the third term which is a constant in the price p^i. This profit is maximized by the same price denoted by p^i_{bo} in (1.19), so loose coupling in Theorem 1 remains valid in all of the models incorporating stockouts.

1.4.3 Analytical Comparisons

We compare the demands (sales probabilities), prices and profits for the retailers under lost sales and backorders in a monopolistic market and a duopolistic market.

Lemma 2. (Lost sales vs. Backorder prices). *a) The lost sales price provides lower and upper bounds for the backorder price:* $p^i_{ls} \leq p^i_{bo} \leq p^i_{ls} + \nu_i d_i$.
b) When a retailer pays $\nu_i d_i$ *back to customers as a backorder penalty and effectively reduces the customer price, the best price to apply in the backorder context turns out to be the same as the lost sales price.*

In our model, the retailer is penalized by d_i for the cost of each backorder. In order to compensate for this penalty, the retailer increases price but not more than $\nu_i d_i$ as shown by Lemma 2.a). If the retailer pays this amount back as in Lemma 2.b), we arrive at the price of the lost sales model, which is the equilibrium price even when the market has retailer preferring and availability preferring customers accepting backorders. Therefore, our chapter focuses on the interesting backorder cases where this amount is an additional cost to the retailer and it is not a payment to the customers.

The choice probability of a retailer favoring customer from retailer i is $\rho^i = \rho^{i,n}_{ret} + \rho^{i,b}_{ret}$, which does not depend on stockout rate ν_i as evident from (1.14)-(1.15). On the contrary, the choice probability of an availability favoring customer from retailer i is $\rho^i_{ava}(\nu_i) := \rho^{i,n}_{ava} + \rho^{i,b}_{ava}$. Hence, $\rho^i_{ava}(\nu_i)$ decreases in ν_i and we can always find $\bar{\nu}^i_{ava}$ such that $\rho^i_{ava}(\nu_i) \geq \rho^i$ for $\nu_i \leq \bar{\nu}^i_{ava}$. Thus, when a retailer's stockout rate is below a threshold, it receives more demand with availability favoring customers rather than retailer favoring customers. Surprisingly, retailer favoring customers do not necessarily increase the demand of retailers. This observation necessitates a refinement of the premise behind increasing customer loyalty. A retailer administering a loyalty program, say by highlighting the ease of repeated purchases from the same retailer or cash-back benefits of loyalty cards, can increase customer loyalty to itself as well as to the other retailer. Even if this program successfully turns all customers from availability favoring to retailer favoring and hence increases loyalty, the retailer administering the program and stocking out infrequently can be worse off in terms of demand. On the other hand, if this retailer stocks out regularly, it is likely to benefit from a loyalty program. Considering the stockout rates helps us to relate them to the demand with and without a loyalty program. The arguments of this paragraph can be extended to analytically compare equilibrium profits as in the next lemma. Superscript "e" on a profit function denotes the equilibrium profit obtained when equilibrium prices are applied. The argument of this function is no longer prices but stockout rates. Note from (1.18) and (1.20) that Π^i_{ret} does not depend on ν_{-i} while Π^i_{ava} does, so we write $\Pi^{i,e}_{ret}(\nu_i)$ and $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i})$.

Lemma 3 (**Retailer vs. Availability Favoring Customers**). *Probabilities: There exists a $\bar{\nu}^i_{ava}$ such that $\rho^i \leq \rho^i_{ava}(\nu_i)$ iff $\nu_i \leq \bar{\nu}^i_{ava}$. Prices: Retailers apply the same equilibrium price in a duopoly in the cases of retailer favoring and availability favoring customers. Also this equilibrium price maximizes the profit in a monopoly. Profits: $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i})$ increases in ν_{-i}. For each ν_i there exists a $\bar{\nu}_{-i}$ such that $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i}) \geq \Pi^{i,e}_{ret}(\nu_i)$ iff $\nu_{-i} \geq \bar{\nu}_{-i}$, and for each ν_{-i} there exists a $\bar{\nu}_i$ such that $\Pi^{i,e}_{ava}(\nu_i) \geq \Pi^{i,e}_{ret}(\nu_i)$ iff $\nu_i \leq \bar{\nu}_i$. Equilibrium profit in the case of availability favoring customers decreases faster than the equilibrium profit in the case of retailer favoring customers as the stockout rate increases but remains low i.e.* $\dfrac{\partial \Pi^{i,e}_{ava}(\nu_i, \nu - i)}{\partial \nu_i} \leq \dfrac{\partial \Pi^{i,e}_{ret}(\nu_i)}{\partial \nu_i} \leq 0$ *for* $\nu_i \leq \bar{\nu}_i$.

Lemma 3 gives retailers information about the usefulness of loyalty programs at the stockout rates (ν_i, ν_{-i}) and shows how our earlier predictions in terms of demands are inherited by profits. A rise in stockout rate ν_{-i} at the competitor increases the profit when customers are availability favoring but keeps it unchanged when customers are retailer favoring. At a low stockout rate at the competitor $\nu_{-i} < \bar{\nu}_{-i}$, the profit is higher when customers are retailer favoring, therefore loyalty programs can be used to transform the market to one with retailer favoring customers. Decreasing its own stockout rate is more beneficial to a retailer with availability favoring customers compared to retailer favoring customers, when $\nu_i \leq \bar{\nu}_{-i}$. Finally, when a retailer's own stockout rate is high, i.e., $\nu_i > \bar{\nu}_i$, it should employ loyalty programs so that customers are retailer favoring, and the retailer in turn makes a large profit. Therefore, WTP-choice model with backorders is useful in determining the stockout rate regions, where loyalty programs are more effective. Sales probabilities under lost sales are compared to those under backorder cases in the next lemma to complete probability comparisons.

Lemma 4 (**Lost sales vs. Retailer and Availability Favoring Customers**). *Probabilities: There exist $\bar{\nu}^i_{ls}$ and $\bar{\nu}^i_{ava}$ such that $\bar{\nu}^i_{ls} \leq \bar{\nu}^i_{ava}$ and $\rho^i \leq \rho^i_{ls}(\nu_i) \leq \rho^i_{ava}(\nu_i)$ for $\nu_i \leq \bar{\nu}^i_{ls}$; $\rho^i_{ls}(\nu_i) \leq \rho^i \leq \rho^i_{ava}(\nu_i)$ for $\bar{\nu}^i_{ls} \leq \nu_i \leq \bar{\nu}^i_{ava}$; $\rho^i_{ls}(\nu_i) \leq \rho^i_{ava}(\nu_i) \leq \rho^i$ for $\bar{\nu}^i_{ava} \leq \nu_i$.*

To compare the prices and profits between the cases of lost sales and backorder, we need a relationship between backorder cost d_i and the other cost parameters. The parameter d_i is not relevant in the lost sales case, so a comparison without restricting d_i is not meaningful. For comparison purposes, we can set the backorder cost equal to the opportunity cost of losing a sale: $d_i = p^i - c_i$. This allows us to write backorder prices and profits by substituting $p^i - c_i$ for d_i in (1.18) and (1.20). The profits for retailer i under retailer favoring case is $\Pi^i_{ret,-d}(p^1, p^2)$ and under availability favoring is $\Pi^i_{ava,-d}(p^1, p^2)$. The profit for a monopolist is $\Pi^i_M(p^i)$ and superscript "$*$" denotes the optimal profit under the optimal price.

Theorem 4. (Profit Comparisons). $\Pi^{i,*}_M \geq \Pi^{i,e}_{ls} \geq \max\{\Pi^{i,e}_{ret,-d}, \Pi^{i,e}_{ava,-d}\}$. *There exist $\bar{\nu}_{-i,-d}$ and $\bar{\nu}_{i,-d}$ such that $\Pi^{i,e}_{ava,-d}(\nu_i, \nu_{-i}) \geq \Pi^{i,e}_{ret,-d}(\nu_i)$ iff $\nu_{-i} \geq \bar{\nu}_{-i,-d}$ and $\Pi^{i,e}_{ava,-d}(\nu_i, \nu_{-i}) \geq \Pi^{i,e}_{ret,-d}(\nu_i)$ iff $\nu_i \leq \bar{\nu}_{i,-d}$.*

From Theorem 4, a monopolist retailer makes the most profit, who is followed by a retailer facing a duopoly under lost sales and then by a retailer facing a duopoly under backorders. The profit realized by a retailer facing backorders depends on fill rate and whether customers are availability favoring or retailer favoring. At first glance, $\Pi^{i,e}_{ls} \geq \max\{\Pi^{i,e}_{ret,-d}, \Pi^{i,e}_{ava,-d}\}$ seems to suggest that lost sales case is better for a retailer than backorder case, however this is true for a specific value of $d_i = p^i - c_i$ that eliminates retailer's margin from a backorder sale. On the other hand, if d_i is independent of p^i, the profit inequality does not hold.

So, Theorem 4 needs to be interpreted carefully. WTP-choice models with lost sales and backorders are used to analyze monopoly and equilibrium prices and profits. They are also useful in modeling customer behaviors such as availability favoring or retailer favoring.

1.5. Estimation with Scanner Data and Numerical Comparisons

We use the maximum (log-) likelihood estimation (MLE) scheme with real-life data pertaining to low-price items, whose choices are likely to be based more on preferences than on utility maximization. We report the mean percentage errors (MPEs) in sales to compare various parameterizations of WTP-choice model with standard and mixed Logit models. Given the scanner data of prices $\{p_n = [p_n^1, \ldots, p_n^M] : n = 1, \ldots, N\}$ and of choices $\{y_n = [y_n^1, \ldots, y_n^M] : n = 1, \ldots, N\}$ along with preferences $\Phi = [\phi_1, \ldots, \phi_S]$ over consideration sets $[\mathfrak{L}_1, \ldots, \mathfrak{L}_S]$, WTPs $W = [W^1, \ldots, W^M]$, $\alpha = [\alpha^1, \ldots, \alpha^M]$ and β, the log-likelihood functions for WTP-choice, standard logit and mixed logit models are

$$\begin{aligned}
L_{wtp}(\delta, \Phi, W) &= \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \rho_n^m(\delta, \Phi, W; p_n, y_n), \\
L_{sl}(\alpha, \beta) &= \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \lambda_n^m(\beta, \alpha; p_n, y_n), \\
L_{ml}(\alpha, F) &= \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \frac{1}{B} \sum_{b=1}^{B} \lambda_n^m(\beta_b, \alpha; p_n, y_n),
\end{aligned}$$

where $y_n^0 = 1 - \sum_m y_n^m$, ρ_n^m is from (1.2-1.3) or (1.4) and λ_n^m is from (1.1), and α_0 is set to zero in Logit model without any loss of generality. For mixed Logit model, we follow the simulated log-likelihood function obtained numerically through simulation (Revelt and Train 1998), where B is the number of draws from the cumulative density function $F(\beta)$ giving $\beta_1, \ldots, \beta_B$.

In Lemma 5 of Appendix A, we establish the concavity of $L_{wtp}(\delta, \Phi, W)$ for given W and concavity of the L_{wtp} for $\llcorner$ distribution in each of the WTP parameters τ_m or a_m.

Without a readily available package to maximize L_{wtp}, we develop a simple search technique for maximization by assuming that either parameters are discrete or are approximated well by discrete values. We identify sets of values for parameters (e.g., δ, ϕ, a_1, a_2, b_1, b_2 for □ WTP-choice model with $M = 2$), whose Cartesian product yield the parameter space. With R (www.r-project.org), we compute the L_{wtp} functions over the parameter space to find the maximizer.

1.5.1 Real-life Data

We have obtained scanner data for *candy melts* from retailer $\mathcal{X}$, which has limited the data exposure. Retailer $\mathcal{X}$ sells various types of candy melts. For estimation with $M = 2$, we consider dark and light (regular) chocolate candy melts. The sales data are weekly, cover about a year and a half, and consist of 14,940 purchases. Retailer $\mathcal{X}$ usually keeps the candy melt prices constant over a week, so the total revenue divided by the total sales for each product in each week is a good indicator of that week's price. The sales and prices for chocolate candy melts are in Figure 1.5, where no-purchases are customers that did not purchase one of the chocolate candy melts under consideration. For $M = 3$, mint chocolate candy melt is considered in addition to dark and light chocolate candy melts. We also use 3 publicly available data sets on 'yogurt', 'ketchup' and 'tuna'. These items have low prices and high purchase frequencies – two factors that may favor WTP-choice model.

Yogurt data (Jain et al. 1994) consists of 2006 observations. For $M = 2$, we focus on two common brand choices of *Dannon* (818 purchases) and *Yoplait* (674 purchases). The remaining 514 customers are put under no-purchase. *Dannon* and *Yoplait* prices respectively range over \$1.9 - \$11.1 and \$0.3 - \$19.3. For $M = 3$, *Hiland* (44 purchases) is the third product with prices over \$2.5 - \$7.6.

Ketchup data (Kim et al. 1995) consists of 4956 observations. For $M = 2$, we focus on *Heinz* (2526 purchases) and *Del Monte* (256 purchases) whose prices respectively range over

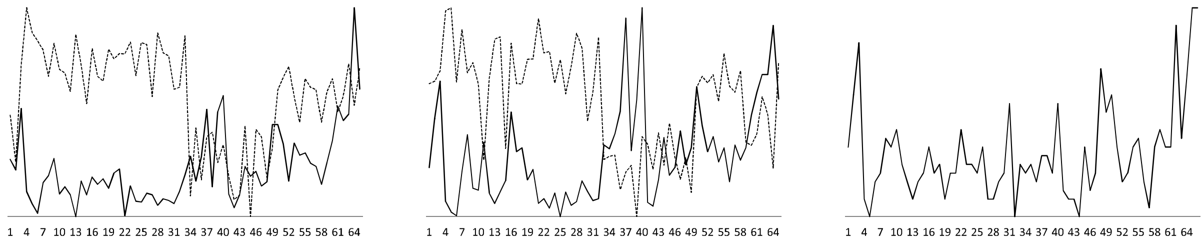


Figure 1.5. Candy melt data for $M = 2$. Left: Weekly sales (–) and prices (- -) for dark chocolate; Middle: Weekly sales (–) and prices (- -) for light chocolate; Right: Weekly no-purchases (–).

\$0.79 - \$1.47 and \$0.89 - \$1.49. For $M = 3$, the *store brand* (1155 purchases) is the third product with prices over \$0.75 - \$0.99.

Canned *Tuna data* (Kim et al. 1995) consists of 13,705 observations. For $M = 2$, we focus on *Sko* (2439 purchases) and *Cosw* (2238 purchases) whose per-pound prices respectively range over \$0.29 - \$0.89 and \$0.19 - \$0.99. For $M = 3$, we also consider *Pw* (1050 purchases) with prices over \$0.17 - \$0.70.

The likelihood function $L_{wtp}(\delta, \phi, a_1, b_1, a_2, b_2)$ for $\square$ WTPs has six parameters. To reduce the dimension of the maximization problem, we can separate the estimation of a_1 and a_2 from the rest. For $\square$ WTPs, the minimum price p_n^m (at which m is sold) is the MLE of a_m, i.e., $a_m^0 := \min\{p_n^m : y_n^m = 1\}$, but then the L_{wtp} has terms such as $\log(p_n^m - a_m^0)$ that become negative infinity when $p_n^m = a_m^0$ for customer i. To avoid this numerical difficulty, we remove those customers who bought product m at the minimum price of a_m^0 and end up with the N (number of purchases) values in Table 1.5.

1.5.2 Likelihood Comparisons of the Choice Models

We consider six different WTP-choice models by setting $\delta = 1$ or $\delta \leq 1$ and parameterize WTPs with a uniform ($\square$), shifted exponential ($\llcorner$) or triangle ($\triangle$) distribution. For example, $\triangle$ WTP and $\delta \leq 1$ make up a special WTP-choice model denoted by $(\triangle, \delta \leq 1)$, and similar notation applies to the others. These distributions have positive supports; $\Box$ and $\llcorner$ distributions have increasing generalized failure rates; $\Box$ distribution can represent a firm uninformed about its customers; $\llcorner$ and $\triangle$ distributions can represent an informed firm that matches the mode of a product's WTP distribution with the reference price of that product. This parameterization flexibility of WTP-choice model is its advantage over Logit model, which allows only double-exponentially distributed utilities. We also set $M = 2$ until prediction accuracy tests.

First, we compare standard Logit model, mixed Logit model and $(\triangle, \delta \leq 1)$ WTP-choice model. The mixed Logit model has the parameters $(\alpha_1, \alpha_2, \mu_\beta, \sigma_\beta)$, where μ_β and σ_β are respectively the mean and standard deviation of the uniformly distributed price coefficient β. $\triangle$ WTP-choice model has parameters $(\delta, \phi, a_1, b_1, m_1, a_2, b_2, m_2)$ but we reduce the number of estimated parameters by fixing $a_i = a_i^0$. Table 1.5 gives the estimates for parameters and the corresponding L values. The number of fixed parameters and their effect on estimation quality is further studied in Table A.1 of Appendix A.

Table 1.5. Log-likelihoods for Standard and Mixed Logit Models and $(\triangle, \delta \leq 1)$ WTP-choice model.

	Standard Logit	Mixed Logit	$(\triangle, \delta \leq 1)$ WTP-choice
Yogurt $N =$ 1,757	$L_{sl} =$ -1,835	$L_{ml} =$ -1,832	$L_{wtp} =$ -1,836
Ketchup $N =$ 4,564	$L_{sl} =$ -4,169	$L_{ml} =$ -4,162	$L_{wtp} =$ -4,152
Candy melt $N =$ 14,125	$L_{sl} =$ -11,498	$L_{ml} =$ -11,496	$L_{wtp} =$ -11,560
Tuna $N =$ 13,332	$L_{sl} =$ -11,143	$L_{ml} =$ -11,143	$L_{wtp} =$ -11,024

From Table 1.5, $\triangle$ WTP-choice model gives a better (higher) L value for ketchup and tuna data compared to mixed Logit model. L values of $-1,832$ and $-1,836$ for yogurt data are very close to each other and the same is true (to a lesser extent) for the candy melt data. L value of the mixed Logit model is smaller than that of WTP-choice model for two data sets. When it is larger, it is so at most by 0.55%. So, $(\triangle, \delta \leq 1)$ WTP-choice model performs as well as the mixed Logit model.

Since WTP-choice model in Table 1.5 has a $\triangle$ WTP distribution, we need to estimate the mode of this distribution. Instead, if we assume a $\square$ WTP distribution (Appendix A has a visual examination), the mode is not required as the support suffices. This reduces the number of parameters by one for each product. Another reduction is obtained by assuming $\delta = 1$, i.e., every customer is interested in buying either one of the products and checks the prices. While reducing the parameters down to (ϕ, b_1, b_2), $(\square, \delta = 1)$ WTP-choice model also reduces the L_{wtp}. Such decreases are reported as Change in % in Table 1.6, where Change in % $:= [L_{wtp}(\triangle, \delta \leq 1) - L_{wtp}(\square, \delta = 1 \text{ or } \llcorner, \delta = 1)]/L_{wtp}(\triangle, \delta \leq 1)$ and $L_{wtp}(\triangle, \delta \leq 1)$ values are from Table 1.5. Similarly, we report Difference in % $:=[L_{ml} - L_{wtp}(\square, \delta = 1 \text{ or } \llcorner, \delta = 1)]/L_{ml}$. Hence, negative values of Change in % and Difference in % indicate a drop from the L values in Table 1.5.

Table 1.6. Changes and differences in log-likelihoods for $\square, \llcorner$ distributed WTPs and $\delta = 1$.

	$(\square, \delta = 1)$ WTP Choice Model			$(\llcorner, \delta = 1)$ WTP Choice Model		
	L_{wtp}	Change in %	Difference in %	L_{wtp}	Change in %	Difference in %
Yogurt	-1,853	-0.93	-1.15	-1,853	-0.93	-1.15
Ketchup	-4,560	-9.83	-9.56	-4,164	-0.22	-0.05
Candy melt	-11,634	-0.64	-1.20	-11,506	0.47	-0.09
Tuna	-13,157	-19.35	-18.07	-11,225	-1.82	-0.74

According to Table 1.6, $(\square, \delta = 1)$ and $(\llcorner, \delta = 1)$ WTP-choice models are outperformed by mixed Logit model. On the other hand, the performance of $(\square, \delta = 1)$ WTP-choice model relative to $(\triangle, \delta \leq 1)$ WTP-choice model suffers significantly with ketchup and tuna data. In other words, WTPs of ketchup and tuna customers resemble $\triangle$ distribution better than $\square$ distribution, and some of these customers are not interested $(\delta \leq 1)$ in buying the particular brands in our data sets. On the contrary, performance of $(\llcorner, \delta = 1)$ WTP-choice model is slightly worse than that of $(\triangle, \delta \leq 1)$ WTP-choice model except for candy melt data where it performs slightly better by 0.47%. The effect of WTP distribution and δ is investigated further in Table A.2 of Appendix A.

Table 1.7 points out WTP-choice model with the highest L_{wtp} for each data set. It specifies WTP-choice model and the estimated parameters. More importantly it compares the log-likelihood of the best WTP-choice model with that of Logit and mixed Logit models. We see that the best WTP-choice model is one of $(\square, \delta = 1)$, $(\triangle, \delta \leq 1)$ or $(\llcorner, \delta \leq 1)$. With yogurt and candy melt data, mixed Logit model performs marginally (at most 0.22%) better. With the other data sets, WTP-choice model performs significantly better (as much as 10.84%). In light of these comparisons, it is safe to propose WTP-choice model as a competitive alternative to Logit models for low-price items.

Table 1.7. Best WTP-choice model versus Logit models.

	Best WTP-choice model				$(L_{wtp} - L_{ml})$ /
	model	L_{wtp}	L_{sl}	L_{ml}	$\lvert L_{wtp}\rvert$ in %
Yogurt	$(\triangle, \delta \leq 1)$	-1,836	-1,835	-1,832	-0.22
Ketchup	$(\square, \delta = 1)$	-3,755	-4,169	-4,162	10.84
Candy melt	$(\llcorner, \delta \leq 1)$	-11,506	-11,498	-11,496	-0.09
Tuna	$(\triangle, \delta \leq 1)$	-11,024	-11,143	-11,143	1.08

1.5.3 Prediction Accuracy Test

We test the accuracy of predictions made with WTP-choice and Logit models. We first split each data set into two sets of equal sizes. One of the sets is referred to as estimation data while the other is test data. In the first step, we estimate the parameters for all models by using the estimation data. Next, we use the estimated parameters to compute the MPE in (expected) sales with all models and the test data. Table 1.8 provides MPEs in % computed as $\left(\sum_{m=1}^{M} \lvert \sum_{n=1}^{N} [\{\hat{\rho}_n^m, \hat{\lambda}_n^m\}] - [\text{actual sales}]^m \rvert\right) / \left(\sum_{m=1}^{M} [\text{actual sales}]^m\right)$ for each test data set, where $\hat{\rho}_n^m$ and $\hat{\lambda}_n^m$ are predicted choice probabilities.

$(\triangle, \delta \leq 1)$ WTP-choice model always predicts the sales more accurately than Logit models for $M = 2$ in Table 1.8. Moreover, $(\triangle, \delta \leq 1)$ WTP-choice model predicts the sales

Table 1.8. MPE in sales for Logit models versus WTP-choice models.

	$M = 2$ Products					$M = 3$ Products		
	Logit models		WTP-choice models			Logit models		WTP-choice
	Standard	Mixed	$\llcorner, \delta \leq 1$	$\square, \delta = 1$	$\triangle, \delta \leq 1$	Standard	Mixed	model $\square, \delta = 1$
Yogurt	2.52	2.31	7.20	2.21	2.17	4.80	4.80	4.90
Ketchup	8.84	8.66	38.86	4.23	7.86	4.78	4.34	4.73
Candy melt	2.30	2.29	19.24	2.26	0.29	1.17	1.17	1.10
Tuna	5.06	5.06	9.84	43.65	2.71	2.84	3.51	11.12

better than ($\llcorner, \delta \leq 1$) and ($\square, \delta = 1$) WTP-choice models, except for ketchup data where ($\square, \delta = 1$) has an error of 4.23% compared with 7.86% for ($\triangle, \delta \leq 1$). For $M = 3$ products, we study consideration sets of size $L = 2$. There standard Logit model performs better with tuna data while mixed Logit and ($\square, \delta = 1$) WTP-Choice model perform better with the other three data sets for predicting sales. In view of our numerical tests with various real-life data, WTP-choice models compare well with Logit models to predict the sales for low-price and high-purchase frequency items.

1.6. Concluding Remarks

This chapter has presented a choice model that captures a customer's WTPs and preferences. WTP-choice model contributes to the choice literature, where customers follow a simple utility satisficing rule and hence have bounded rationality. WTP-choice model has several desirable properties: it explicitly captures the sequence of products considered and requires limited information; the choice probabilities are the same when prices are learnt first or last; and it does not have the IIA property.

Under competitive pricing with independent WTPs and no inventory consideration, we show that retailers are loosely coupled – equilibrium profits are coupled but prices are not. Loose coupling facilitates the implementation of the equilibrium prices on a competitive market. However, when the WTPs are dependent or the preferences are dependent on the prices, or customers are heterogeneous, loose coupling fails and the retailers must consider the competitor's price while deciding on own prices. Furthermore, setting price at par with the competitor is not necessarily the equilibrium strategy even when retailers have identical products and costs. We also show that price cycles exist in competitive markets where the WTPs of the products are dependent, and provide conditions to eliminate price cycles to guarantee the existence of a price-pair equilibrium.

The sequential decision making structure of WTP-choice models help us study stockouts, under which loose coupling remains valid. We derive equilibrium profits and prices for the case of lost sales, the case of backorders further specified by retailer favoring and availability favoring customers. Eventually, we compare the monopoly and price competition results under lost-sales and backorders. In particular, we discuss the effect of retailer favoring behavior (as opposed to availability favoring) on sales probabilities and equilibrium profits depending on stockout rates, and we connect this effect to loyalty programs.

For empirical validation, we compare the fit and prediction accuracy of standard Logit, mixed Logit and WTP-choice models by using real-life data. Our real-life data consist of products with low price and high purchase frequency for which customers are likely to use utility satisficing (WTP-choice) rather than maximizing (Logit). WTP-choice models usually perform better than or on par with Logit models. WTP-choice model can also be used to estimate WTP of customers as the model is flexible and does not assume a specific distribution of WTP.

WTP-choice model has a simple satisficing assumption for the customers, is designed to be versatile due to general and dependent WTP distributions and a sequential framework accommodating stockouts, and yields simple pricing formulas for independent WTPs. It can serve as a basis for interesting future studies such as further empirical studies and more dynamic and detailed pricing applications.

CHAPTER 2

CONTINGENT SOURCING UNDER SUPPLY DISRUPTION AND COMPETITION

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2.1. Introduction

Supply chain disruption risk is becoming an increasingly important topic of study in operations management. Disruptions in supply chains make suppliers unable to fulfill the product quantities ordered by manufacturers/buyers. Failure to meet demand can be caused by bottlenecks in production or supply processes and natural disasters such as power outages, terrorist attacks, natural hazards, etc. Supply chain disruptions may cause suppliers to default in meeting the manufacturer's orders. Modern business operations such as outsourcing and external procurement are becoming increasingly common, but they tend to make supply chains highly inter-dependent. With such dependence, a default on the part of an upstream supplier leads to supply disruptions downstream. Therefore, one of the biggest challenges faced by supply chain managers in today's globalized and highly uncertain business environment is to proactively and efficiently prepare for possible disruptions that may affect complex supply chain networks (Gurnani et al. 2012). In this chapter, we only consider disruptions that cause a supplier to not fulfil the order altogether. Cases of partial fulfilment (Li et al. 2012, 2013 and Dada et al. 2007) or late fulfilment (Dolgui and Ould-Louly 2002) are not discussed.

The literature on supply chain disruptions is very diversified. Snyder et al. (2010) provide an excellent review of the literature on supply chain disruption management. They discuss nearly 150 scholarly papers on topics including evaluation of supply disruptions, strategic decisions, sourcing decisions, contracts and incentives, inventory, and facility location. The supply chain disruption literature on which our work is based can be classified into four streams: (1) price-dependent demand, (2) competition among buyers, (3) default by an unreliable supplier, and (4) contingent sourcing strategy to overcome supply disruptions.

The first line of the literature focuses on inventory decisions with price-dependent demands. Early on, most of the operations management literature dealt with pricing in inventory/capacity management focused on a single product with perfectly reliable supply. Whitin

(1955) and Mills (1959, 1962) were among the first who considered endogenous prices in inventory/capacity models. Dada et al. (2007) and Li et al. (2012) study sourcing and pricing decisions of a firm ordering from several suppliers and facing a price-dependent demand. They show that for a firm, *cost is the order qualifier while reliability is the order winner* in choosing a supplier. Ha and Tong (2008) study two competing firms under contracts with suppliers and facing a demand that is linear in price. The market demand can be low or high. Shou et al. (2009) also use a price-dependent linear demand to study management of supply chains under disruption.

The second stream studies competing suppliers and buyers exposed to supply disruptions. Shou et al. (2009) discuss two competing supply chains subject to supply uncertainties. The retailers engage in a Cournot competition by determining the quantities to be ordered from their exclusive suppliers. They examine the decisions of the suppliers and the retailers at three different levels: operational, design, and strategic. They find that supply chain coordination may or may not result in positive gains for the supply chain, depending on the extent of the supply risk. Li et al. (2010) examine the sourcing strategy of a retailer and the pricing strategies of two suppliers in a supply chain facing supply disruptions. They use the spot market as a perfectly reliable contingent supplier and characterize the sourcing strategy of the retailer in both centralized and decentralized systems. Tang and Kouvelis (2011) study the benefits from supplier diversification for two dual-sourcing competing buyers. The authors conclude that buyers should never choose to use a common supplier, because the increased correlation between the delivered quantities leads to a decrease in the buyers' profits.

Wang et al. (2009) compare the effectiveness of dual sourcing and reliability improvement strategies. They show that a combined strategy of contingent dual sourcing and reliability improvement can provide significant value if suppliers are very unreliable and/or capacity is low relative to demand. Tomlin (2009a) study how supply learning affects sourcing and inventory policies when firms adopt contingent dual sourcing or single sourcing. He analyzes a Bayesian model (via distribution updating) of "supply learning" to investigate how supply learning affects both sourcing and inventory decisions in single sourcing and dual sourcing models.

We consider a supply chain with two suppliers, U and R, and two competing buyers, C and S, selling the same products in the market. The competition between the buyers is modeled as a Cournot quantity game. Supplier U is Unreliable and Supplier R is Reliable but more expensive. This setting is also used by Tomlin (2006) and Chopra et al. (2007); see also Kazaz (2004) and Tomlin and Snyder (2006). We assume that Firm C places an order with Supplier U first, and will place an emergency order with Supplier R if Supplier U cannot deliver the order. For expositional brevity, we refer to this as a Contingent Dual Sourcing Strategy (CDSS). Firm S purchases only from Supplier R. We refer to this as a Sole Sourcing Strategy, or SSS for short. We characterize the equilibrium quantity and the expected profit for each manufacturer under different cases, and obtain important managerial insights.

There have been a number of real-life instances of CDSS reported in the literature. For example, in response to the air traffic disruption resulting from 9/11, Chrysler temporarily shipped components by ground from the U.S. to their Dodge Ram assembly plant in Mexico (Tomlin 2006). The primary benefit of CDSS over maintaining safety stock is that the cost is incurred only in the event of a supply disruption. Although CDSS has been used by many firms, there is a lack of research on the impact of such a strategy on supply chains. Is it always cost effective? Under what conditions is it a dominant strategy to manage supply disruptions? How does CDSS affect firms' decisions under competition? We add to the literature on CDSS by investigating the strategy in a duopoly setting. Although, the timing of supply disruptions is unpredictable, firms have control over procurement times. C buyer can place his order before, after, or simultaneously with another buyer.

The remainder of this chapter is organized as follows. In section 2.2, the duopoly model is introduced and formulated. In section 2.3, we analyze various possible games in the duopoly setting under different cases. For each Case, we obtain the equilibrium order quantities and expected profits for both buyers. We also obtain some properties of the equilibrium orders as well as the associated expected profits. In section 2.4, we compare the equilibria in the games studied in section 2.3. Numerical computations to get further insights are presented in section 2.5. In section 2.6, we study two extensions of the model studied in section 2.3. In section 2.6.1 and section 2.6.2 we discuss the equilibrium long-run sourcing strategies of two asymmetric and symmetric firms that can choose between SSS and CDSS. We study the impact of capacity reservation by the CDSS firm to secure supply from Supplier R in section 2.6.3. Finally, section 2.7 summarizes the main contributions of our work and suggests future research directions.

2.2. The Model

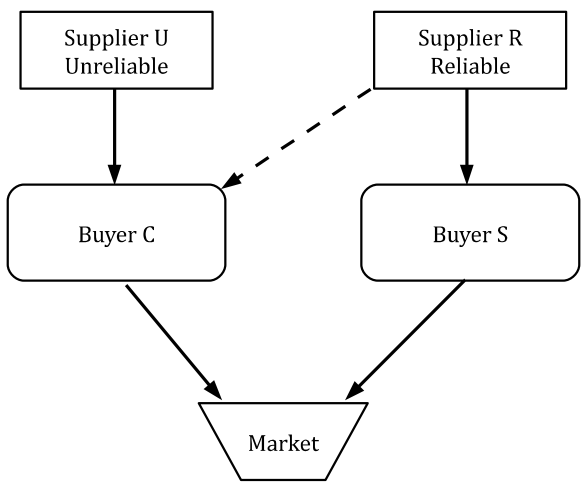


Figure 2.1. Sourcing Model

To investigate the impact of supply disruptions on competing buyers, we consider a supply chain as depicted in Figure 2.1. There are two buyers C and S who procure parts from suppliers U and R. C and S process these parts into substitutable products to be sold at the same price in the market to end consumers. The product has a short life cycle and is sold in a single selling season. Supplier U is unreliable and Supplier R is reliable, and the unit costs charged by them are c_1 and c_2 respectively, where $c_2 > c_1 > 0$ for regular orders. In normal situations (*without supply disruptions*), C buys products from Supplier U. If Supplier U defaults, C places an emergency order with Supplier R at a unit cost of c_3 where $c_3 \geq c_2$ on account of S being Supplier R's preferred customer. A relevant question is whether the scenario we model in this section can arise if sourcing strategies of firms is endogenous. To answer this, we provide in section 2.6.1 an example of two asymmetric firms C and S with $c_3 > c_2$ in which Firm C chooses CDSS and Firm S chooses SSS when the reliability of Supplier U is at a medium level. Of course, here we study the case in the general parameter settings of $c_3 \geq c_2$.

We let the binary random variable X, taking values in $\{0, 1\}$, denote the supply state of Supplier U. When $X = 1$, Supplier U can deliver all that is ordered, and when $X = 0$, he can deliver nothing. We assume that α denotes the probability that $X = 1$. Supplier R on the other hand can always deliver whatever is ordered.

We model the manufacturers to engage in a quantity competition. C orders Q_1 from Supplier U and S orders Q_2 from Supplier R. If Supplier U defaults, then C orders Q_{e0} from Supplier R, otherwise none is ordered, i.e. $Q_{e1} = 0$. The price p of the products is determined by a linear demand function $p(S) = a - S$, where S is the total product quantity delivered to buyers C and S, and $a > 0$ denotes the potential market size. We assume that a is sufficiently large to ensure that $p > 0$; as shown in the next section, we must assume that $a > 3c_3 - 2c_2 + c_1$. The linear demand function substantially simplifies the analysis, and has been extensively used in the literature; see e.g., Ha and Tong (2008), Shou and Huang (2009), and Li et al. (2013), and references therein.

In order to compare the equilibrium quantities ordered and the expected profits for C and S, we study a benchmark supply chain when Supplier R is the only supplier and both C and S order from it.

2.2.1 The Benchmark Profits for C and S under SSS

When both buyers order from Supplier R, there are three possible ordering sequences: (i) both order simultaneously, (ii) C orders first, and (iii) S orders first.

In Case (i) , the buyers play a Nash game. The optimal order quantities for C and S are $(a + c_2 - 2c_3)/3$ and $(a + c_3 - 2c_2)/3$, respectively. Accordingly, the maximum profits for C and S are $(a + c_2 - 2c_3)^2/9$ and $(a + c_3 - 2c_2)^2/9$, respectively.

In Case (ii), when C orders before S does, it becomes a Stackelberg game with C as the leader and S as the follower. The equilibrium order quantity and profit for C are $(a + c_2 - 2c_3)/2$ and $(a + c_2 - 2c_3)^2/8$, respectively. The equilibrium order quantity and the profit for S are $(a - 3c_2 + 2c_3)/4$ and $(a - 3c_2 + 2c_3)^2/16$, respectively.

Case (iii) with S ordering first is symmetric to Case (ii), and so the equilibrium order quantity and the equilibrium profit, respectively, for C are $(a-3c_3+2c_2)/4$ and $(a - 3c_3 + 2c_2)^2/16$ and for S are $(a + c_3 - 2c_2)/2$ and $(a + c_3 - 2c_2)^2/8$.

The maximum possible profits for C and S over all three cases when ordering from Supplier R are $(a + c_2 - 2c_3)^2/8 =: \Pi_C^0$ and $(a + c_3 - 2c_2)^2/8 =: \Pi_S^0$, respectively. These will serve as the benchmark profits for assessing the benefits for C and S, respectively, in all games considered in this chapter.

2.3. Analysis of the Games with Dual Sourcing

There are a number of events taking place in our model – C orders, S orders, the supply state realizes, and C places an emergency order if Supplier U is disrupted. Depending on the occurrence of these events, there arise seven different cases shown in Figures 2.2 to 2.8. We let $Q_1^j, Q_2^j, Q_e^j, \Pi_C^j, \Pi_S^j, S^j$ denote C's order quantity, S's order quantity, C's emergency order quantity, C's expected profit, S's expected profit, and the total market supply in Case j in equilibrium, $j = 1, 2, \ldots, 7$. We also define the notation Q_{e0} as the emergency order when

$x = 0$ and $Q_{e1} = 0$ when $x = 1$. For the multistage games under consideration, the concepts we use are feedback Nash and feedback Stackelberg equilibria as defined, e.g., in Basar and Olsder (1999) and Bensoussan et. al (2014). These can be obtained by a backward induction procedure, and are time consistent or sub-game perfect. We analyze the cases one by one and find the quantities ordered and the expected profits in equilibrium in each Case. We also discuss the sensitivity of the results with respect to the model parameters in each Case. We obtain managerial insights indicating how the results change with respect to the model parameters in each Case.

2.3.1 Case 1

The sequence of events in this Case is shown in Figure 2.2.

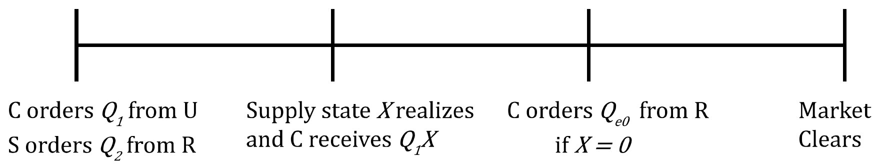


Figure 2.2. Sequence of events for Case 1

In this Case, C and S simultaneously order Q_1 and Q_2 from Suppliers U and R, respectively in the first stage. Then the supply state X realizes and C receives the quantity Q_1X from Supplier U and S receives the quantity Q_2 from Supplier R. In the next stage, C places an emergency order Q_{e0} if $X = 0$. After that, the market clears and the profits of C and S are realized.

The game played is a multi-stage game in which C places an emergency order from Supplier R, after both C and S have ordered Q_1 and Q_2 simultaneously from Suppliers U and R, respectively, after the supply state X is realized. We use backward induction to obtain the equilibrium solution. That is, C's emergency order quantity response will be given as a feedback function $q_e(Q_1, Q_2, x)$, where x is the realization of X. If Supplier U does not default, i.e., $x = 1$, then clearly $q_e(Q_1, Q_2, 1) = 0$. However, when $x = 0$, C will maximize his profit to obtain $q_e(Q_1, Q_2, 0)$, i.e., $\max_{q_e}[(a - Q_2 - q_e - c_3)\, q_e]$. By solving this, we obtain the best response of buyer C given Q_1 and Q_2 as $q_e(Q_1, Q_2, 0) = \dfrac{(a - Q_2 - c_3)}{2}$. Thus, the entire feedback policy is

$$q_e(Q_1, Q_2, x) = \begin{cases} Q_{e0} = \dfrac{a - Q_2 - c_3}{2} & \text{if } x = 0, \\ Q_{e1} = 0 & \text{if } x = 1. \end{cases} \tag{2.1}$$

Next we solve the Nash game between C and S, knowing C's emergency order quantity reaction function. That is, C and S obtain Q_1 and Q_2 simultaneously by maximizing their respective expected profits. In view of (2.1), therefore, we have the following simultaneous maximization problems:

$$\max_{Q_1} \left[\alpha\,(a - Q_1 - Q_2 - c_1)\, Q_1 + (1-\alpha) \left(\frac{a - Q_2 - c_3}{2} \right)^2 \right], \tag{2.2}$$

$$\max_{Q_2} \left[\alpha\,(a - Q_1 - Q_2 - c_2)\, Q_2 + (1-\alpha) \left(a - \frac{a - Q_2 - c_3}{2} - Q_2 - c_2 \right) Q_2 \right]. \tag{2.3}$$

Solving the first-order condition gives

$$Q_1^{1*} = \frac{(1+\alpha)(a - 2c_1) + 2c_2 - (1-\alpha)c_3}{2\,(\alpha+2)} \text{ and } Q_2^{1*} = \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{\alpha + 2}. \tag{2.4}$$

These are indeed the equilibrium order quantities since the objective functions (2.2) and (2.3) are jointly strictly concave in Q_1 and Q_2. The equilibrium in Case 1 can now be expressed as the triple $(Q_1^{1*}, Q_2^{1*}, Q_e^{1*})$, where Q_e^{1*} is the random variable $Q_e^{1*} = q_e(Q_1^{1*}, Q_2^{1*}, X)$. Inserting $Q_e^{1*} = 0$ when $X = 1$ and $Q_{e0} = \dfrac{(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3}{2\,(\alpha+2)}$ into the objective functions (2.2) and (2.3), we obtain the equilibrium expected profits for C and S, respectively, as

$$E\left(\Pi_C^1\right) = \frac{\alpha\left[(1+\alpha)(a-2c_1) + 2c_2 - (1-\alpha)c_3\right]^2 + (1-\alpha)\left[(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3\right]^2}{4\,(\alpha+2)^2},$$

$$E\left(\Pi_S^1\right) = \frac{(1+\alpha)\left(a + \alpha c_1 - 2c_2 + (1-\alpha)c_3\right)^2}{2(\alpha+2)^2}.$$

The expected total market output is

$$E\left(S^1\right) = \alpha\left(Q_1^{1*} + Q_2^{1*}\right) + (1-\alpha)\left(Q_e^{1*} + Q_2^{1*}\right) = \frac{(3+\alpha)a - \alpha(1+\alpha)c_1 - 2c_2 - (1-\alpha^2)c_3}{2\,(\alpha+2)}.$$

Proposition 1. *Q_1^{1*} increases in α and c_2 and decreases in c_1 and c_3; Q_2^{1*} decreases in α and c_2 and increases in c_1 and c_3; and Q_e^{1*} increases in α and decreases in c_1, c_2 and c_3, almost surely.*

Proposition 1 says that when c_2 increases, Supplier U has a higher cost advantage over supplier R. So, C increases the quantity ordered from supplier U. When c_1 increases, the cost advantage for C reduces and he buys less from supplier U. An increase in supplier U's reliability α means that C has a higher chance of realizing the cost advantage over S, and therefore, C buys more and S buys less from Supplier R.

Proposition 2. *In the equilibrium of Case 1, the expected total market output is decreasing in c_1, c_2, and c_3, and increasing in α if $2c_2 + (\alpha^2 + 4\alpha + 1)\, c_3 > a + (\alpha^2 + 4\alpha + 2)\, c_1$ and non-increasing otherwise.*

The expected market price in Case 1 is $E\left(p^1\right) = a - E\left(S^1\right)$, and it is straightforward to obtain the results about the expected market price from the expected total market supply.

2.3.2 Case 2

The sequence of events in this Case is shown in Figure 2.3.

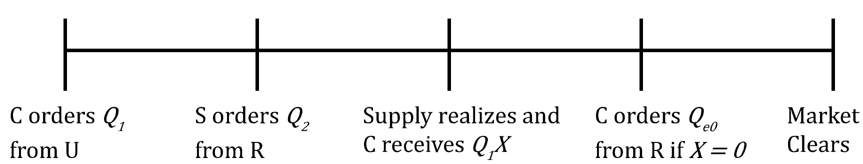


Figure 2.3. Sequence of events for Case 2

Here, C orders Q_1 first from Supplier U and then S orders Q_2 from Supplier R. Then the supply state X realizes and C receives Q_1X from Supplier U and S receives Q_2 from Supplier R. Following this, C places an emergency order Q_e with Supplier R depending on the realization of X. After that, the market clears and the profits of C and S are realized.

In the first stage, C leads and S follows in placing the orders Q_1 and Q_2, respectively. In the second stage, S leads with his order Q_2 and C follows by his emergency order Q_e. Therefore, C's order quantity response is the same feedback function $q_e(Q_1, Q_2, x)$ as given in (2.1). Anticipating this and knowing C's order Q_1, S maximizes his expected profit. Thus, S's problem is:

$$\max_{Q_2} \left[\alpha \left(a - Q_1 - Q_2 - c_2 \right) Q_2 + (1-\alpha) \left(\frac{a - Q_2 + c_3 - 2c_2}{2} \right) Q_2 \right]. \tag{2.5}$$

Using the first-order condition, we obtain S's best response function

$$q_2(Q_1) = \frac{(1+\alpha)a - 2c_2 + (1-\alpha)c_3 - 2\alpha Q_1}{2(1+\alpha)}, \tag{2.6}$$

which C uses in the first stage to obtain his order Q_1. For this, C maximizes his expected profit:

$$\max_{Q_1} \left[\alpha \left(a - Q_1 - q_2(Q_1) - c_1 \right) Q_1 + (1-\alpha) \left(\frac{a - q_2(Q_1) - c_3}{2} \right)^2 \right], \tag{2.7}$$

and obtains

$$Q_1^{2*} = \frac{a(\alpha+1)(\alpha+3) - 4(\alpha+1)^2 c_1 + 2\alpha(c_2 + c_3) + 6c_2 + 3\alpha^2 c_3 - 5c_3}{2(4 + 3\alpha + \alpha^2)}. \tag{2.8}$$

Plugging (2.8) in (2.6), we get

$$Q_2^{2*} = \frac{2\left(a + (\alpha+1)\alpha c_1 - (\alpha+2)c_2 + \alpha^2(-c_3) + c_3\right)}{4 + 3\alpha + \alpha^2}. \tag{2.9}$$

These are indeed the equilibrium order quantities since the objective functions (2.5) and (2.7) are jointly strictly concave in Q_1 and Q_2. Thus, the equilibrium triple in Case 2 is given by $(Q_1^{2*}, Q_2^{2*}, Q_e^{2*})$, where $Q_e^{2*} = q_e\left(Q_1^{2*}, Q_2^{2*}, 0\right)$ with q_e as defined in (2.1). In particular, when $X = 0$, the emergency order quantity

$$q_e\left(Q_1^{2*}, Q_2^{2*}, 0\right) = Q_{e0}^2 = \frac{a(\alpha+1)(\alpha+2) + \alpha(-2(\alpha+1)c_1 + 2c_2 + (\alpha-3)c_3) + 4c_2 - 6c_3}{2(4 + 3\alpha + \alpha^2)}.$$

Inserting (2.8) and (2.9) into the objective functions (2.5) and (2.7), we obtain the equilibrium expected profits for C and S, respectively, as

$$\begin{aligned} E\left(\Pi_C^2\right) &= \frac{1}{4(4+3\alpha+\alpha^2)}\big[\alpha^2\big(a^2+a(6c_3-8c_1)+8c_1^2-4c_1(c_2+c_3)+c_3(4c_2-3c_3)\big)+\\ &\quad 2\alpha\big(a^2-a(3c_1-2c_2+c_3)+2c_1^2+c_1(5c_3-6c_2)+4c_3(c_2-c_3)\big)+\\ &\quad 2\alpha^3(c_3-c_1)(a-2c_1+c_3)+(a+2c_2-3c_3)^2\big],\\ E\left(\Pi_S^2\right) &= \frac{2(\alpha+1)\left(a+(\alpha+1)\alpha c_1-(\alpha+2)c_2+\alpha^2(-c_3)+c_3\right)^2}{(4+3\alpha+\alpha^2)^2}. \end{aligned}$$

The expected total market output $E\left(S^2\right)$ is given by

$$E\left(S^2\right)=Q_2^{2*}+\alpha Q_1^{2*}+(1-\alpha)\,Q_{e0}^{2*}=\frac{a(\alpha(\alpha+2)+3)+(\alpha+1)^2((\alpha-1)c_3-\alpha c_1)-2c_2}{4+3\alpha+\alpha^2}.$$

Proposition 3. *The expected total market supply $E\left(S^2\right)$ is decreasing in c_1, c_2, and c_3, and increasing in α if*

$$6c_2-c_3+a(-1+\alpha(2+\alpha))-c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha)))+\alpha(4c_2+c_3(10+\alpha(16+\alpha(6+\alpha))))>0.$$

2.3.3 Case 3

The sequence of events in this Case is shown in Figure 2.4.

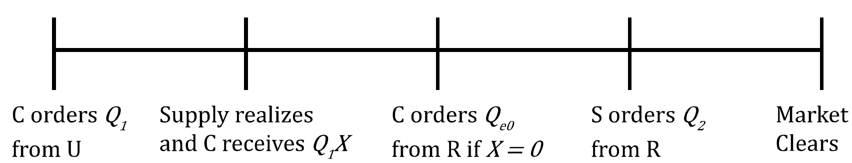


Figure 2.4. Sequence of events for Case 3

In this Case, C first orders Q_1 and then the supply state X realizes and C receives Q_1X from Supplier U. Following this, C places an emergency order Q_{e0} with Supplier R if $X=0$.

After that, S orders Q_2 from Supplier R. Finally, the market clears and the profits of C and S are realized.

The game involves a multi-stage game in which S follows by deciding Q_2 given C's two-stage decision of ordering Q_1 and then Q_{e0} if $X = 0$. Therefore, S's profit maximization problem is:

$$\max_{Q_2} \left[(a - Q_1 x - Q_e(Q_1, x) - Q_2 - c_2)\, Q_2 \right], \tag{2.10}$$

where $Q_e(Q_1, x)$ is the emergency order quantity by C when $X = x$. We can solve (2.10) to obtain the reaction function of S:

$$q_2(Q_1, Q_e(Q_1, x)) = \frac{a - Q_1 x - Q_e(Q_1, x) - c_2}{2}. \tag{2.11}$$

The emergency order $Q_{e1} = 0$ when $x = 1$, so we only need to solve for Q_1 and Q_{e0}. This can be done by solving C's expected profit maximization problem:

$$\max_{Q_1, Q_{e0}} \left[\alpha \left(a - Q_1 - \frac{a - Q_1 - c_2}{2} - c_1 \right) Q_1 + (1 - \alpha) \left(a - Q_{e0} - \frac{a - Q_{e0} - c_2}{2} - c_3 \right) Q_{e0} \right]. \tag{2.12}$$

This gives

$$Q_{e0}^{3*} = \frac{a + c_2 - 2c_3}{2} \text{ and } Q_1^{3*} = \frac{a + c_2 - 2c_1}{2}. \tag{2.13}$$

Moreover, we can express C's emergency order feedback policy as

$$Q_e^{3*}(Q_1, x) = \begin{cases} \dfrac{a + c_2 - 2c_3}{2} & \text{if } x = 0, \\ 0 & \text{if } x = 1. \end{cases} \tag{2.14}$$

Substituting (2.13) and (2.14) in (2.11), we get S's equilibrium equilibrium order quantity Q_2^{3*} as

$$Q_2^{3*}(x) = \begin{cases} \dfrac{a - 3c_2 + 2c_3}{4} & \text{if } x = 0, \\ \dfrac{a - 3c_2 + 2c_1}{4} & \text{if } x = 1. \end{cases} \tag{2.15}$$

Inserting (2.13), (2.14), and (2.15) into the objective functions (2.10) and (2.12), we obtain the equilibrium expected profits for C and S as

$$E\left(\Pi_C^3\right) = \alpha\frac{(a+c_2-2c_1)^2}{8} + (1-\alpha)\frac{(a+c_2-2c_3)^2}{8},$$

$$E\left(\Pi_S^3\right) = \alpha\frac{(a-3c_2+2c_1)^2}{16} + (1-\alpha)\frac{(a-3c_2+2c_3)^2}{16}.$$

It is easy to see that $E\left(\Pi_C^3\right) > E\left(\Pi_S^3\right)$ when $c_2 = c_3$. Also, $E\left(\Pi_C^3\right) < E\left(\Pi_S^3\right)$ for large values of c_3 and small α. The expected total market output is

$$\begin{aligned} E\left(S^3\right) &= \alpha\left(\frac{a+c_2-2c_1}{2} + \frac{a-3c_2+2c_1}{4}\right) + (1-\alpha)\left(\frac{a+c_2-2c_3}{2} + \frac{a-3c_2+2c_3}{4}\right) \\ &= \frac{3a-2\alpha c_1-c_2-2(1-\alpha)c_3}{4}. \end{aligned}$$

Proposition 4. *At the equilibrium of Case 3, the expected total market output $E\left(S^3\right)$ increases in α and decreases in c_1, c_2, and c_3. The expected market price $E\left(p^4\right)$ decreases in α and increases in c_1, c_2, and c_3.*

2.3.4 Case 4

The sequence of events in this Case is shown in Figure 2.5.

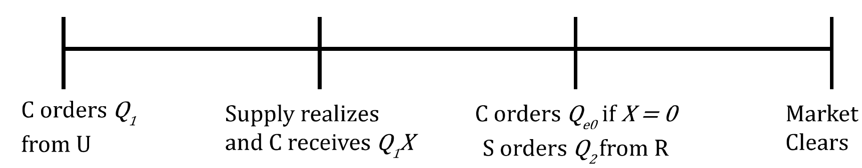


Figure 2.5. Sequence of events for Case 4

Thus in this Case, C orders Q_1 first, then supply state X realizes, and C receives Q_1 if Supplier U does not default. Next, C and S simultaneously order Q_e and Q_2 from Supplier R, respectively. After that, the market clears and the profits of C and S are realized.

In the second stage, C and S order Q_e and Q_2 simultaneously from Supplier R, after C has ordered Q_1 from Supplier U in the first stage and the supply state X is realized at the end of the first stage. Note that if Supplier U defaults, i.e., $X = 0$, the game at the second stage is a simple quantity competition game, where C and S order $(a + c_2 - 2c_3)/3$ and $(a + c_3 - 2c_2)/3$ from Supplier R, respectively. When $X = 1$, C does not place an emergency order and S's profit maximization problem is:

$$\max_{Q_2} (a - Q_1 - Q_2 - c_2)\, Q_2. \tag{2.16}$$

The first-order condition gives

$$q_2(Q_1, 1) = \frac{a - Q_1 - c_2}{2}. \tag{2.17}$$

Therefore, we obtain C's and S's order quantity responses in the equilibrium as

$$q_e^{4*}(Q_1, x) = \begin{cases} \dfrac{a + c_2 - 2c_3}{3} & \text{if } x = 0, \\ 0 & \text{if } x = 1, \end{cases} \tag{2.18}$$

$$Q_2^{4*}(Q_1, x) = \begin{cases} \dfrac{a - 2c_2 + c_3}{3} & \text{if } x = 0, \\ \dfrac{a - Q_1 - c_2}{2} & \text{if } x = 1. \end{cases} \tag{2.19}$$

Next, we solve the game at the first stage in which C anticipates his own and S's order quantity responses in the second stage given by (2.18) and (2.19), respectively, and orders Q_1 that maximizes his expected profit, i.e.,

$$\max_{Q_1} \left[\alpha \left(\frac{a + c_2 - 2c_1 - Q_1}{2} \right) Q_1 + (1 - \alpha) \left(\frac{a + c_2 - 2c_3}{3} \right)^2 \right]. \tag{2.20}$$

From the first-order condition, we solve for Q_1^{4*}, C's equilibrium order quantity in stage 1, as

$$Q_1^{4*} = \frac{a + c_2 - 2c_1}{2}. \tag{2.21}$$

Substituting (2.21) into (2.19), we have $Q_2^{4*} = \dfrac{a - 3c_2 + 2c_1}{4}$, if $x = 1$. Using the equilibrium order quantities, we can now derive the expected profits for C and S as

$$\begin{aligned} E\left(\Pi_C^4\right) &= \alpha \frac{(a + c_2 - 2c_1)^2}{8} + (1 - \alpha) \frac{(a + c_2 - 2c_3)^2}{9}, \\ E\left(\Pi_S^4\right) &= \alpha \frac{(a - 3c_2 + 2c_1)^2}{16} + (1 - \alpha) \frac{(a - 2c_2 + c_3)^2}{9}. \end{aligned}$$

It is easy to see that $E\left(\Pi_C^4\right) > E\left(\Pi_S^4\right)$ when $c_2 = c_3$. Also, $E\left(\Pi_C^4\right) < E\left(\Pi_S^4\right)$ for large values of c_3 and small α. The expected total market output is

$$\begin{aligned} E\left(S^4\right) &= \alpha\left(\frac{a+c_2-2c_1}{2}+\frac{a-3c_2+2c_1}{4}\right)+(1-\alpha)\left(\frac{a+c_2-2c_3}{3}+\frac{a-2c_2+c_3}{3}\right), \\ &= \alpha\left(\frac{3a-c_2-2c_1}{4}\right)+(1-\alpha)\left(\frac{2a-c_2-c_3}{3}\right). \end{aligned}$$

Proposition 5. *In the equilibrium of Case 4, the expected total market output* $E\left(S^4\right)$ *is increasing in* α*, and decreasing in* c_1*,* c_2*, and* c_3*. Consequently, the expected market price* $E\left(p^4\right)$ *is decreasing in* α *but increasing in* c_1*,* c_2*, and* c_3*.*

2.3.5 Case 5

The sequence of events in this Case is shown in Figure 2.6.

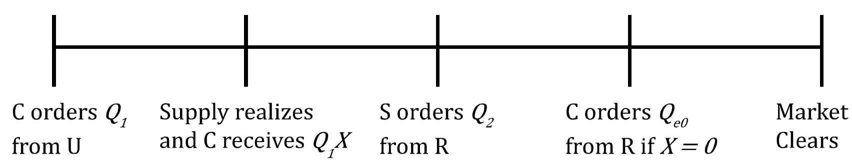


Figure 2.6. Sequence of events for Case 5

Here, C orders Q_1 first and then the supply state X realizes, and then C receives Q_1X from Supplier U. After that S orders Q_2 from Supplier R. Following this, C places an emergency order Q_{e0} with Supplier R if $X = 0$. Finally, the market clears and the profits of C and S are realized.

The game has two stages. In the first stage, C leads and S follows in placing the orders Q_1 and Q_2, respectively. In the second stage, C follows and S leads when they place orders Q_e and Q_2, respectively. Therefore, in the second stage, C's order quantity response will be the feedback function $q_e(Q_1, Q_2, x)$ given in (2.1).

Next, we solve the game between C and S in the second stage where S anticipates C's emergency order $Q_e(Q_1, Q_2, X)$ and maximizes his expected profit. From (2.1), we obtain

S's expected profit maximization problem as

$$\max_{Q_2} E\left[\left(a - Q_1 x - Q_e(Q_1, Q_2, X) - c_2\right) Q_2\right]. \tag{2.22}$$

Using (2.1), we obtain the order quantity response for S using the first-order condition

$$q_2(Q_1, X) = \frac{a - c_2}{2} - \frac{Q_1 X}{X + 1}. \tag{2.23}$$

From this and (2.1), C's emergency order quantity Q_{e0}^{4*} when $X = 0$ is

$$Q_{e0}^{5*} = \frac{a + c_2 - 2c_3}{4}. \tag{2.24}$$

Now, we solve the game in the first stage where C factors in S's order quantity $Q_2 = q_2(Q_1, X)$ given by (2.23), and orders Q_1 from Supplier U that maximizes his expected profit, i.e.,

$$\max_{Q_1} \left[\alpha \left(\frac{a - Q_1 - 2c_1 + c_2}{2}\right) Q_1 + (1 - \alpha)\left(\frac{a + c_2 - 2c_3}{4}\right)^2\right]. \tag{2.25}$$

Solving this, gives

$$Q_1^{5*} = \frac{a + c_2 - 2c_1}{2}. \tag{2.26}$$

Plugging (2.26) in (2.23), we obtain

$$Q_2^{5*}(x) = \begin{cases} \dfrac{a - c_2}{2} & \text{if } x = 0, \\ \dfrac{a - 3c_2 + 2c_1}{4} & \text{if } x = 1. \end{cases} \tag{2.27}$$

The expected profits for C and S are

$$\begin{aligned} E\left(\Pi_C^5\right) &= \alpha \frac{(a + c_2 - 2c_1)^2}{8} + (1 - \alpha)\frac{(a + c_2 - 2c_3)^2}{16}, \\ E\left(\Pi_S^5\right) &= \alpha \frac{(a - 3c_2 + 2c_1)^2}{16} + (1 - \alpha)\frac{(a - c_2)^2}{8}. \end{aligned}$$

A direct analytical comparison between the expected profit of C and S is not possible. So, we resort to numerical analysis in section 2.5 to compare the profits. The expected total market output is

$$\begin{aligned}
E\left(S^5\right) &= \alpha\left(\frac{a+c_2-2c_1}{2}+\frac{a-3c_2+2c_1}{4}\right)+(1-\alpha)\left(\frac{a+c_2-2c_3}{4}+\frac{a-c_2}{2}\right)\\
&= \alpha\left(\frac{3a-c_2-2c_1}{4}\right)+(1-\alpha)\left(\frac{3a-c_2-2c_3}{4}\right)\\
&= \frac{3a-2\alpha c_1-c_2-2(1-\alpha)c_3}{4}.
\end{aligned}$$

Proposition 6. *In the equilibrium, $E\left(S^5\right)$ increases in α and decreases in c_1, c_2, and c_3. The expected market price $E\left(p^5\right)$ decreases in α and increases in c_1, c_2, and c_3.*

2.3.6 Case 6

The sequence of events in this Case is shown in Figure 2.7.

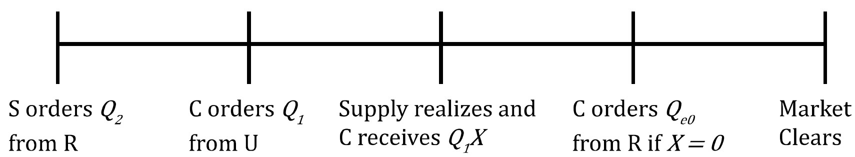


Figure 2.7. Sequence of events in Case 6

This results in a multi-stage game where S leads by ordering Q_2 and C follows with orders Q_1 in the first stage. In the second stage, C's order quantity response function will be the same feedback function $q_e(Q_1, Q_2, x)$ given by (2.1), since the game is identical to the game in Case 1. And we are left with the problem of solving only the first stage of the game where C orders Q_1 by maximizing his expected profit:

$$\max_{Q_1}\left[\alpha\left(a-Q_1-Q_2-c_1\right)Q_1+(1-\alpha)\frac{\left(a-Q_2-c_3\right)^2}{4}\right]. \tag{2.28}$$

From the first-order condition, we have

$$q_1(Q_2)=\frac{a-Q_2-c_1}{2}. \tag{2.29}$$

In the first stage, S leads and orders Q_2 from Supplier R by anticipating C's order quantity responses Q_1 and Q_e. Therefore, S's expected profit maximization problem in view of (2.1) and (2.29) is given as

$$\max_{Q_2} \left[\alpha \frac{(a - Q_2 + c_1 - 2c_2)}{2} Q_2 + (1-\alpha) \frac{(a - Q_2 - 2c_2 + c_3)}{2} Q_2 \right]. \tag{2.30}$$

Using the first-order condition, we obtain the equilibrium order quantity

$$Q_2^{6*} = \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{2}.x \tag{2.31}$$

From (2.1), (2.29), and (2.31), we obtain

$$Q_1^{6*} = \frac{a - (2+\alpha)c_1 + 2c_2 - (1-\alpha)c_3}{4}, \tag{2.32}$$

$$q_e^{6*}(x) = \begin{cases} \dfrac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4} & \text{if } x = 0, \\ 0 & \text{if } x = 1. \end{cases} \tag{2.33}$$

Using these order quantities, we find the expected profits for C and S as

$$\begin{aligned} E\left(\Pi_C^6\right) &= \alpha \left(\frac{a - (\alpha+2)c_1 + 2c_2 - (1-\alpha)c_3}{4} \right)^2 + (1-\alpha) \left(\frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4} \right)^2 \\ &= \frac{1}{16} \left((a + 2c_2 - 3c_3)^2 + 2(3a - 2c_1 + 6c_2 - 7c_3)(-c_1 + c_3)\alpha + 5(c_1 - c_3)^2 \alpha^2 \right), \\ E\left(\Pi_S^6\right) &= \left(\frac{a - c_2 - \alpha c_2 + \alpha c_1}{2} \right). \\ &\qquad \left\{ \alpha \left(\frac{a - 3c_2 + \alpha c_2 - \alpha c_1 + 2c_1}{4} \right) + (1-\alpha) \left(\frac{a - c_2 + \alpha c_2 - \alpha c_1}{4} \right) \right\} \\ &= \frac{1}{8} (a + \alpha c_1 - 2c_2 + (1-\alpha)c_3)^2. \end{aligned}$$

Here, a direct comparison of C and S's expected profit is not straightforward. So, we resort to numerical analysis in section 2.5 to compare the profits. The expected total market output is

$$\begin{aligned} E\left(S^6\right) &= \alpha \left(\frac{a - (2+\alpha)c_1 + 2c_2 - (1-\alpha)c_3}{4} + \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{2} \right) \\ &\quad + (1-\alpha) \left(\frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4} + \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{2} \right) \\ &= \frac{1}{4} \left(3a - \alpha c_1 - 2c_2 - (1-\alpha)c_3 \right). \end{aligned}$$

Proposition 7. *In the equilibrium of Case 6, the* $E\left(S^6\right)$ *increases in* α *and decreases in* c_1*,* c_2*, and* c_3*.* p^6 *decreases in* α *and increases in* c_1*,* c_2*, and* c_3*.*

2.3.7 Case 7

The sequence of events in Case 7 is shown in Figure 2.8.

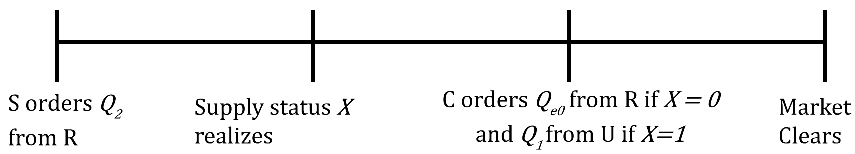


Figure 2.8. Sequence of events for Case 7

In the last Case, S orders Q_2 first from Supplier R and then the supply state X is realized. Following this, C orders Q_1 from Supplier U if $X = 1$ or places an emergency order of Q_e with Supplier R if $X = 0$. After that, the market clears and the profits of C and S are realized.

In the multi-stage game played here, S leads by ordering Q_2 from Supplier R, and C follows with orders Q_1 from Supplier U if $x = 1$, or Q_{e0} from Supplier R, when $x = 0$. C's order quantity response will be the same feedback function $q_e(Q_1, Q_2, x)$ as given by (2.1). Next, we solve the game when the realized supply state is $x = 1$. C, being the follower, orders Q_1 from Supplier U to maximize his expected profit, i.e.,

$$\max_{Q_1}[(a - Q_1 - Q_2 - c_1)\, Q_1]. \tag{2.34}$$

From the first-order condition, we obtain

$$q_1(Q_2) = \frac{a - Q_2 - c_1}{2}. \tag{2.35}$$

S anticipates C's order quantities (2.1) and (2.35), and orders Q_2 from Supplier R that maximizes his expected profit:

$$\max_{Q_2}\left[\alpha\frac{(a - Q_2 + c_1 - 2c_2)}{2}Q_2 + (1-\alpha)\,\frac{(a - Q_2 - 2c_2 + c_3)}{2}Q_2\right]. \tag{2.36}$$

The first-order condition gives S's equilibrium order quantity Q_2^{7*} for the first-stage game as

$$Q_2^{7*} = \frac{a - 2c_2 + \alpha c_1 + (1-\alpha)c_3}{2}. \tag{2.37}$$

From (2.1), (2.35), and (2.37), we obtain the equilibrium order quantities

$$Q_1^{7*} = \frac{a - (2+\alpha)c_1 + 2c_2 - (1-\alpha)c_3}{4} \text{ and } Q_e^{7*} = \frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4}.$$

The expected profits of C and S are

$$\begin{aligned}
E\left(\Pi_C^7\right) &= \alpha \left(\frac{a - (\alpha+2)c_1 + 2c_2 - (1-\alpha)c_3}{4}\right)^2 + (1-\alpha)\left(\frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4}\right)^2 \\
&= \frac{1}{16}\left((a + 2c_2 - 3c_3)^2 + 2(3a - 2c_1 + 6c_2 - 7c_3)(-c_1 + c_3)\alpha + 5(c_1 - c_3)^2\alpha^2\right), \\
E\left(\Pi_S^7\right) &= \left(\frac{a - c_2 - \alpha c_2 + \alpha c_1}{2}\right) \cdot \\
&\qquad \left\{\alpha\left(\frac{a - 3c_2 + \alpha c_2 - \alpha c_1 + 2c_1}{4}\right) + (1-\alpha)\left(\frac{a - c_2 + \alpha c_2 - \alpha c_1}{4}\right)\right\} \\
&= \frac{1}{8}(a + \alpha c_1 - 2c_2 + (1-\alpha)c_3)^2.
\end{aligned}$$

The expected total market output is identical to that in Case 6. Since $E\left(S^7\right) = E\left(S^6\right)$, all other equilibrium results in Case 7 are identical to those in Case 6.

2.4. Equilibrium Profit Comparisons

In the previous section, we derived explicit expressions for the equilibrium expected profits of C and S in all cases. In section 2.2.1, we obtained the benchmark profit. In this section, we compare the expected profits of C and S in all 7 cases and also to the benchmark profit.

Proposition 8. *(i) Expected profits for C satisfy:* $E\left(\Pi_C^2\right) > E\left(\Pi_C^1\right) > E\left(\Pi_C^6\right) = E\left(\Pi_C^7\right)$, $E\left(\Pi_C^3\right) > E\left(\Pi_C^4\right) > E\left(\Pi_C^5\right)$, *and* $E\left(\Pi_C^5\right) > E\left(\Pi_C^1\right)$.
(ii) Expected profits for S satisfy: $E\left(\Pi_S^2\right) < E\left(\Pi_S^1\right) < E\left(\Pi_S^6\right) = E\left(\Pi_S^7\right)$, $E\left(\Pi_S^3\right) < E\left(\Pi_S^4\right) < E\left(\Pi_S^5\right)$, *and* $E\left(\Pi_S^1\right) > E\left(\Pi_S^4\right)$.

(iii) The expected profits of C satisfy $E(\Pi_C^i) \le \Pi_C^0$ *for* $i \ne 3$ *and* $E(\Pi_C^3) > \Pi_C^0 = (a + c_2 - 2c_3)^2/8$, *the benchmark profit. The expected profits of S in all cases satisfy* $E(\Pi_S^i) \le \Pi_C^0$. *Finally,* $E\left(\Pi_C^3\right) > E\left(\Pi_S^3\right)$.

As we go from Case 3 to 5 , the time when S places orders is later and later. This confers an advantage to S, and consequently his expected profit increases and the expected profit of C decreases as shown in Proposition 8 (i) and (ii).

In Proposition 8 (iii) we compare the expected profits of C with the benchmark profit Π_C^0. We observe that only in Case 3, C has a higher profit than its benchmark profit. The intuitive explanation is that purchasing from an unreliable supplier is better for someone with first mover advantage and when the supply state is realized before the competitor orders. Similarly, the equilibrium expected profits of S are less than the benchmark profit in all cases. Therefore, the profit of C is always more than the profit of S in Case 3. This is further corroborated by the numerical computations in section 2.5.

Proposition 9. *If* $\alpha \ge 0.5$ *and* $c_2 = c_3$, *then* $E\left(\Pi_C^5\right) > E\left(\Pi_S^5\right)$.

2.4.1 Comparative Statics

The comparative statics of the equilibrium expected profits of C and S are summarized in Table 2.1. We see that the equilibrium expected profits for both C and S are monotone in α, c_1, and c_3. C's profit is not monotone with c_2 and S's profit is decreasing with c_2. In the absence of monotonicity of $E(\Pi_C^i)$ with c_2, we conjecture that when the supply from 1 is highly reliable, $E(\Pi_C^i)$ is increasing in c_2, and when the supply from 1 is highly unreliable, $E(\Pi_C^i)$ is decreasing in c_2. The intuitive explanation for this conjecture relies on the trade-off between the supply costs c_1, c_2, and c_3 to C and S and the reliability α of Supplier U. With a higher reliability of Supplier U, the chance of order fulfilment is higher, i.e., C purchases at the cheaper price c_1 from Supplier U with a higher chance. Subsequently, C earns more in the market due to his cost advantage over S since $c_2 \ge c_1$.

Table 2.1. Comparative statics of equilibrium expected profits under cases $i = 1 \ldots 7$. Here, '$\uparrow$' indicates increasing, '$\downarrow$' indicates decreasing and '$\updownarrow$' indicates non-monotonicity.

Profit $\rightarrow$	$E\left(\Pi_C^i\right)$			$E\left(\Pi_S^i\right)$		
Parameter $\rightarrow$	α	c_1, c_3	c_2	α	c_1, c_3	c_2
	$\uparrow$	$\downarrow$	$\updownarrow$	$\downarrow$	$\uparrow$	$\downarrow$

We now summarize the comparative statics of the equilibrium order quantities for C and S in all 7 cases. The most interesting takeaway from Table 2.2 is the impact of reliability on the order quantities. We see that Q_1^{i*} and Q_{e0}^{i*} are increasing in α in cases 1 and 2, where C orders from Supplier U before or at the same time as S does from Supplier R. In cases 6 and 7, S orders from Supplier R before C does from suppliers U and R. Therefore, both Q_1^{i*} and Q_{e0}^{i*} are decreasing in α in cases 6 and 7. Similarly, we see that Q_2^{i*} is decreasing in α in cases 1 and 2 and increasing in α for cases 6 and 7, respectively. We also see from Table 2.2 that the quantity ordered by a firm is always non-increasing in its own procurement costs and non-decreasing in its competitor's procurement costs.

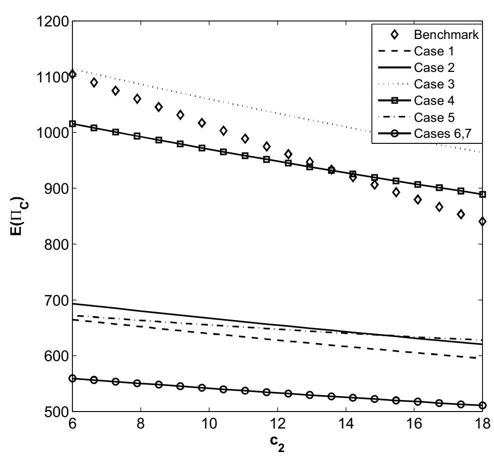
Table 2.2. Comparative statics of equilibrium order quantities. Here, $\uparrow$ indicates increasing, $\downarrow$ indicates decreasing, and $\updownarrow$ indicates unchanging. $\uparrow$ ($\updownarrow$) and $\downarrow$ ($\updownarrow$) indicates increasing and decreasing in c_1, respectively and unchanging in c_3. $\updownarrow$ ($\downarrow$) indicates unchanging in c_1 and decreasing in c_3.

Order Quantity $\rightarrow$	Q_1^{i*}			Q_2^{i*}			Q_{e0}^{i*}		
Parameter $\rightarrow$	α	$c_1(c_3)$	c_2	α	$c_1(c_3)$	c_2	α	$c_1(c_3)$	c_2
Case 1	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Case 2	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Case 3	$\updownarrow$	$\downarrow$ ($\updownarrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\updownarrow$	$\updownarrow$ ($\downarrow$)	$\uparrow$
Case 4	$\updownarrow$	$\downarrow$ ($\updownarrow$)	$\uparrow$	$\updownarrow$	$\uparrow$	$\downarrow$	$\updownarrow$	$\updownarrow$ ($\downarrow$)	$\uparrow$
Case 5	$\updownarrow$	$\downarrow$ ($\updownarrow$)	$\uparrow$	$\uparrow$	$\uparrow$ ($\updownarrow$)	$\downarrow$	$\updownarrow$	$\updownarrow$ ($\downarrow$)	$\uparrow$
Case 6	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
Case 7	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$

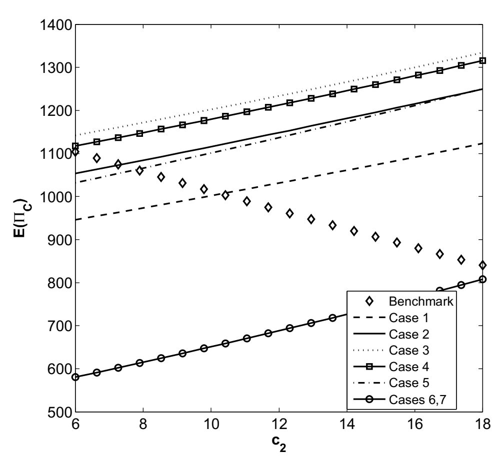
2.5. Computational Analysis

In this section, we present the results obtained from computations that confirm as well as add to the conclusions drawn in section 2.3. We compute the expected equilibrium profits for both C and S in all cases by varying c_1, c_2, c_3, and α. Since the equilibrium profits of C as well as S are identical in Cases 6 and 7, we analyze them together as 'Cases 6, 7'. The profits of C and S are monotone in the same direction with both c_1 and c_3, so we assume $c_3 = c_2$ when analyzing the impacts of c_1 and c_2 on the equilibrium expected profits of C and S. In particular, we first illustrate the impact of combinations of α, c_1 and c_2 on the equilibrium expected profits of C and S and on the equilibrium total order quantities in all cases. We let $a = 100$ in all computations.

2.5.1 Impact of Supplier Costs and Reliability on Profit of C



(a) $\alpha = 0.2$.

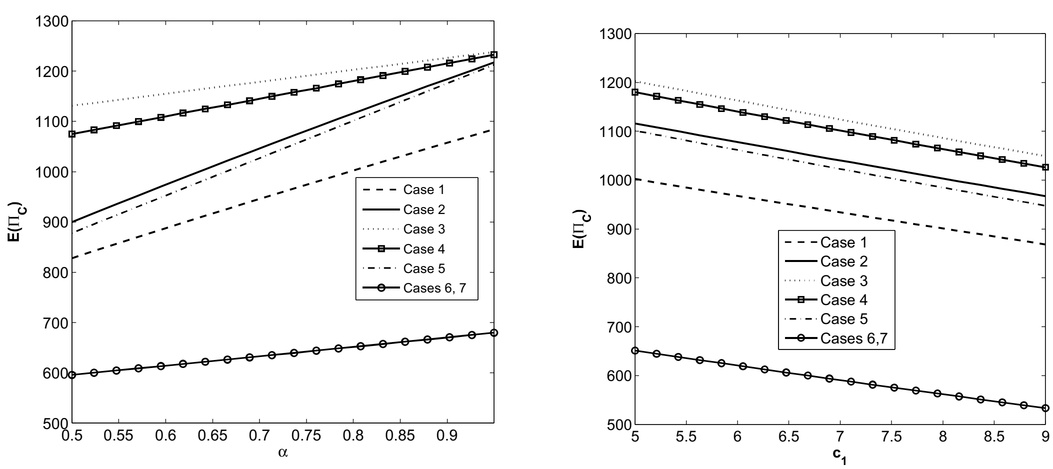


(b) $\alpha = 0.8$.

Figure 2.9. Variation of $E(\Pi_C)$ with respect to c_2 and α.

Figures 2.9(a) and 2.9(b) show the equilibrium expected profit of C with respect to c_2 for low and high value of α, respectively. For computing $E(\Pi_C)$, we let $c_1 = 5$ and vary $c_2 \in [6, 18]$. First we see in Figure 2.9(a) that when the supplier is highly unreliable, say when $\alpha = 0.2$,

$E(\Pi_C)$ is decreasing in c_2. On the other hand, we see in Figure 2.9(b) that when the supplier is highly reliable (say $\alpha = 0.8$) then an increase in the cost for S leads to an increase in the profit of C. This observation is in line with the conjecture proposed in section 2.4. This dependence of the monotonicity of $E(\Pi_C)$ with c_2 on α can be explained as follows: When Supplier U is highly reliable, there is less of a chance that C will order from Supplier R at the higher cost $c_3 \geq c_2$. This increases C's profitability. We also see from Figure 9 that the profits for C satisfy $E\left(\Pi_C^3\right) > E\left(\Pi_C^4\right) > E\left(\Pi_C^{2,5}\right) > E\left(\Pi_C^1\right) > E\left(\Pi_C^6\right) = E\left(\Pi_C^7\right)$. This is in line with Proposition 9(i). This shows that adopting CDSS is beneficial for C as it increases his profits.



(a) Variation with α. (b) Variation with c_1.

Figure 2.10. Variation of $E(\Pi_C)$ with respect to α and the wholesale cost c_1.

Figure 2.10(a) shows the equilibrium expected profit of C in all cases with α. We let $c_1 = 5, c_2 = 10$, and vary $\alpha \in [0.5, 0.95]$. We observe that the profit of C always increases in α, which is also shown in Table 2.1. The cause of this increase in profit of C is due to the increase in reliability of Supplier U, which gives C a cost advantage over S as Supplier U is cheaper than 2. Therefore, the expected equilibrium profit of C increases in α. Figure 2.10(b) shows how c_1 affects the profit of C. In these numerical computations, we fix $\alpha = 0.8,\ c_2 = 10$, and vary $c_1 \in [5,9]$. We see that the profit of C decreases with an increase in c_1, which is also seen in Table 2.1. This decrease in the profit of C is due to the decrease in the benefit of ordering from the cheaper supplier. Accordingly, C orders less from Supplier U and the profit of C decreases. The orders between the expected profits of C under different cases remain intact in Figures 2.10(a) and 2.10(b), as the order of profits in Figure 2.9(a) and 2.9(b).

2.5.2 Impact of Supplier Costs and Reliability on Profit of S

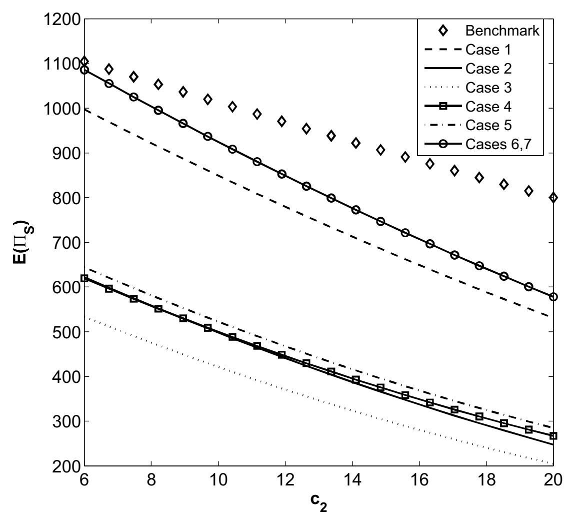
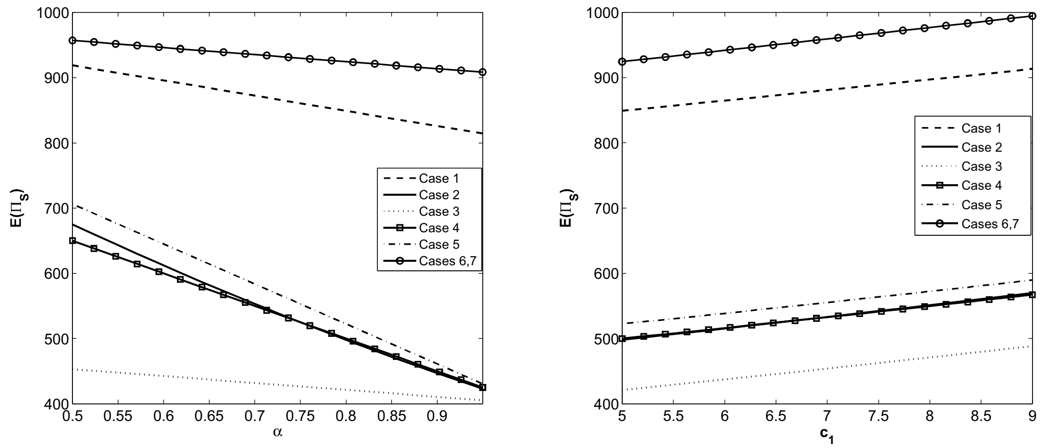


Figure 2.11. Variation of equilibrium profit $E(\Pi_S)$ with respect to the cost c_2.

To study the variation of the equilibrium expected profit of S with c_2, we fix $c_1 = 5$, $\alpha = 0.8$ and vary $c_2 \in [6,20]$. Figure 2.11 shows that $E(\Pi_S)$ decreases in c_2, in all cases. This confirms with the comparative statics reported in Table 2.1. $E(\Pi_S)$ in all cases satisfy: $E(\Pi_S^3) < E(\Pi_S^2) < E(\Pi_S^4) < E(\Pi_S^5) < E(\Pi_S^1) < E(\Pi_S^6) = E(\Pi_S^7)$. We find that $E(\Pi_S)$ increases from Case 3 through Case 6, as the procurement decision of S from Supplier R is later and later in time. We also see that the profits of S in all cases are lower than the benchmark profit, even when he decides before C, e.g., S orders from Supplier R before C does from Supplier U in cases 6 and 7.



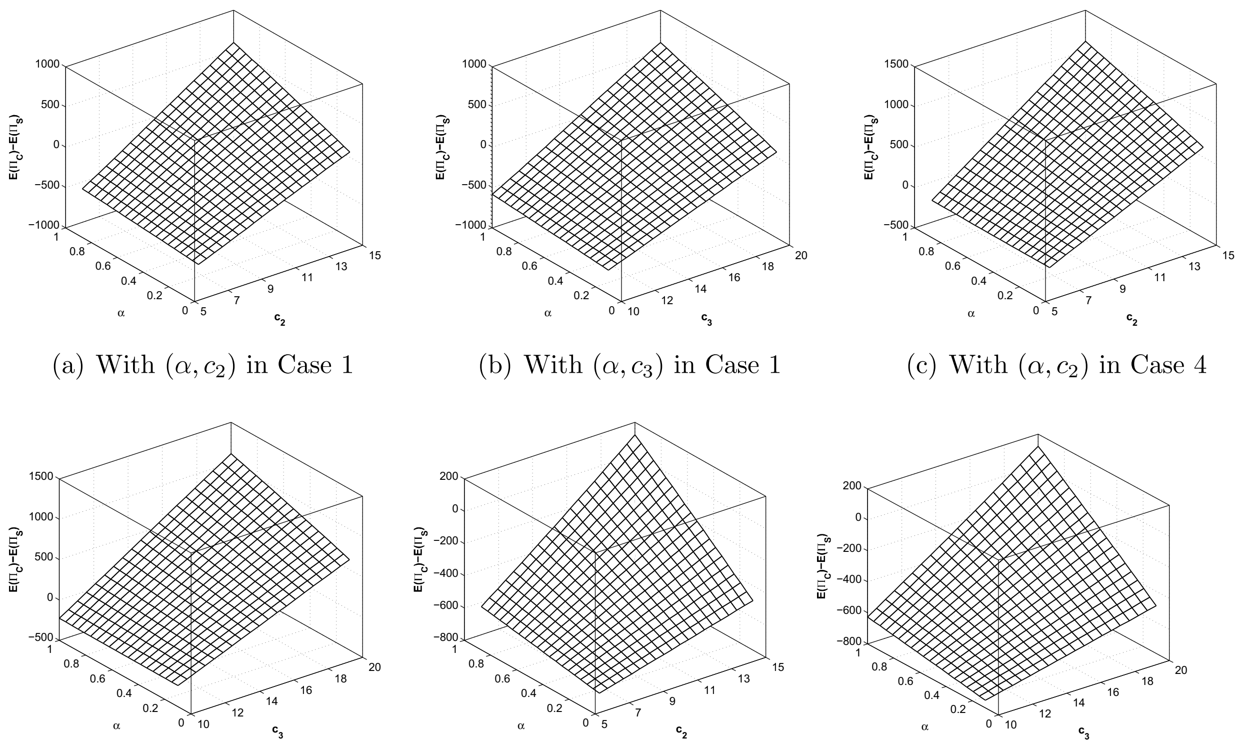
(a) Variation with α. (b) Variation with c_1.

Figure 2.12. Variation of equilibrium profit of S with respect to reliability α and the wholesale cost c_1.

Figure 2.12(a) shows the impact of the reliability of Supplier U on the profits of S. We fix $c_1 = 5$, $c_2 = 10$ and vary $\alpha \in [0.5, 0.95]$. We observe from Figure 2.12(a) that $E(\Pi_S)$ decreases in α in all cases. Further, to study the impact of Supplier U's cost c_1 on $E(\Pi_S)$, we fix $\alpha = 0.8$, $c_2 = 10$ and vary $c_1 \in [5, 9]$. From Figure 2.12(b), we see that $E(\Pi_S)$ increases in c_1 in all cases. We also see that the equilibrium expected profit of S satisfy: $E\left(\Pi_S^3\right) < E\left(\Pi_S^4\right) \leq E\left(\Pi_S^2\right) < E\left(\Pi_S^5\right) < E\left(\Pi_S^1\right) < E\left(\Pi_S^6\right) = E\left(\Pi_S^7\right)$.

2.5.3 Impact of Contingent Sourcing on Profits

Now we study the impact of CDSS strategy on the equilibrium expected profits of C and S. To be able to study the impact of CDSS on the profits of C and S, we compute the difference between the profits of C and S, who adopt CDSS and SSS strategies, respectively. The difference between the profits of C and S is $E\left(\Pi_C^i\right) - E\left(\Pi_S^i\right)$. Therefore, CDSS does better than SSS only if $E(\Pi_C^i) - E(\Pi_S^i) > 0$. Note that, in section 2.4 we established $E\left(\Pi_C^3\right) > E\left(\Pi_S^3\right)$. Therefore, CDSS is better than SSS when C and S operate in a situation identical to Case 3. We also compare cases $i \in \{1,4,6\}$ using computational analysis. For these computations, we use $c_1 = 5$, $\alpha \in [0.1, 0.95]$, $c_2 \in [6, 15]$, and $c_3 \in [10, 20]$.



(a) With (α, c_2) in Case 1 (b) With (α, c_3) in Case 1 (c) With (α, c_2) in Case 4

(d) With (α, c_3) in Case 4 (e) With (α, c_2) in Case 6 (f) With (α, c_3) in Case 6

Figure 2.13. $E(\Pi_C^i) - E(\Pi_S^i)$ with respect to $(\alpha, c_2)\,,(\alpha, c_3)$ under cases 1,4, and 6.

In Figures 2.13(a)-(f), we see that there is no clear winner between CDSS and SSS. If α is low and c_2 is not too large compared to c_1, then SSS is better than CDSS for S and worse for C. If α is high and c_2 is larger than c_1, then CDSS is better than SSS for C and worse for S.

2.5.4 Impact of Reliability Level on Total Market Output

To study the impact of reliability on the total market output $E\left(S^i\right)$, we set $c_1 = 5$, $c_2 = 10$ and vary $\alpha \in [0.1, 1.0]$. Figure 2.14 shows the variation of $E\left(S^i\right)$ with respect to α. We see that $E\left(S^i\right)$ has no clear monotonicity in all cases. $E\left(S^i\right)$ is not necessarily increasing with

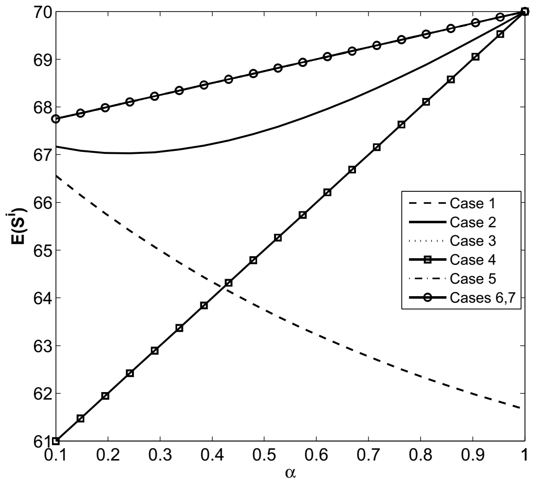


Figure 2.14. $E\left(S^i\right)$ with α

the reliability of Supplier U. Surprisingly, in Case 1 it is strictly decreasing. The expected total market output in Case 1 satisfies Proposition 2, i.e., when $2c_2 + \left(\alpha^2 + 4\alpha + 1\right) c_3 > a + \left(\alpha^2 + 4\alpha + 2\right) c_1$, then $E\left(S^1\right)$ is decreasing with α. Therefore, Figure 2.14 validates the proposition using the specific parameter values we chose for these computations.

2.6. Extensions

In this section we discuss three important extensions. In the first two, we extend the model presented in section 2.3 to study endogenous sourcing strategies for asymmetric and symmetric firms in the market. The asymmetric firms C and S differ in the sense that $c_3 > c_2$. In section 2.6.1 we provide examples that show that, depending on the parameters, the firms should choose different strategies or the same strategies in equilibrium. Moreover, these examples indicate that depending on the cost difference, if the reliability of Supplier U is high, medium, or low, both firms choose CDSS, Firm C chooses CDSS and Firm S chooses SSS, or both firms choose SSS, respectively. The last choice results interestingly from a prisoner's dilemma. In section 2.6.2, the Firms C and S are symmetric with $c_3 = c_2$. Now both firms choose CDSS if Supplier U is highly reliable. Otherwise and once again, both are in a prisoner's dilemma and choose SSS. In section 2.6.3 we study the impact of capacity reservation by C with the reliable supplier R on the equilibrium order quantities.

2.6.1 Endogenous Sourcing Strategy: Asymmetric Firms

Thus far we have studied the firms that choose order quantities in the short run with specified sourcing strategies. However, in the long run, firms can decide their sourcing strategies by maximizing their respective profits. So here we study two asymmetric firms C and S, interested in choosing their optimal sourcing strategies from CDSS or SSS and deciding simultaneously their order quantities, as studied in section 2.3.1. There are three cases to consider: (a) C chooses CDSS and S chooses SSS; (b) both choose SSS; and (c) both choose CDSS. The profits of C and S in Case (a) are given in section 2.3.1. In Case (b), C and S order from Supplier R, and the equilibrium order quantity is $\frac{a-c_2}{3}$. The expected profit for each firm is $\frac{(a-c_2)^2}{9}$. Case (c) is a two-stage game between the firms. In the first stage C and S order from Supplier U, and in the second stage they order from Supplier R if there is disruption. Accordingly, the equilibrium order quantity from Supplier U is $\frac{a-c_1}{3}$ and from Supplier R is $\frac{a-c_3}{3}$. The expected profit for each firm is $\frac{\alpha(a-c_1)^2+(1-\alpha)(a-c_3)^2}{9}$. Since $c_3 > c_1$, this profit increases with α. Therefore, there is a threshold $\alpha_r = \frac{(2a-c_2-c_3)(c_3-c_2)}{(2a-c_1-c_2)(c_2-c_1)+(2a-c_2-c_3)(c_3-c_2)}$, such that the profit in Case (c) is higher than the profit in Case (b) with $\alpha > \alpha_r$. Furthermore, when $c_2 = c_3$, the profit in Case (c) is always higher than in Case (b), and vice-versa if $c_1 = c_2$.

A reason for asymmetry may be due to the location of these firms in relation to Supplier R. Say, for example, that Firm S is located nearer to Supplier R than Firm C is. Then it is reasonable to assume that $c_2 < c_3$. This difference in the costs could also arise if S has a closer relationship with Supplier R. Furthermore, for convenience of not having too many cases to deal with, we assume that the firms are symmetric in all other aspects.

		Buyer S	
	Expected Profit	CDSS	SSS
Buyer C	CDSS	**944.4, 982.2**	926.2, 889.0
	SSS	849.0, 1002.3	711.1, 900.0

Both firms choose CDSS when reliability is high with $\alpha = 0.8$.

		Buyer S	
	Expected Profit	CDSS	SSS
Buyer C	CDSS	886.1, 961.7	**742.5, 980.0**
	SSS	895.7, 887.7	711.7, 900.0

Asymmetric sourcing equilibrium when $\alpha = 0.6$.

		Buyer S	
	Expected Profit	CDSS	SSS
Buyer C	CDSS	798.6, 930.8	473.6, 1120.6
	SSS	962.4, 704.1	**711.1, 900.0**

Both firms choose SSS when reliability is low with $\alpha = 0.3$. This game is the classical ‘prisoner’s dilemma’.

Figure 2.15. Equilibrium sourcing strategies for buyers with different reliability of supplier U.

In Figure 2.15 we present the payoffs for C and S in a 2x2 matrix when they choose CDSS or SSS with different reliability levels of the supplier U. We set $a = 100$, $c_1 = 5$, $c_2 = 10$, $c_3 = 20$ and vary $\alpha \in \{0.3, 0.6, 0.8\}$. We call these levels low, medium and high, respectively, for the purpose of this discussion. We observe that in the equilibrium, both choose CDSS when the the reliability is high and choose SSS, by playing a prisoner's dilemma game when the reliability is low. However, at the medium reliability level, C chooses CDSS and S chooses SSS. It is this last setting that motivates the formulation of our model in section 2.3 where we assume Firms C and S to follow CDSS and SSS, respectively.

2.6.2 Endogenous Sourcing Strategy: Symmetric Firms

We now study symmetric firms and show that they, as expected, choose the same sourcing strategy in equilibrium as well as identify the situations where they would choose CDSS or SSS. At a low reliability level of Supplier U, we find that C and S play a prisoner's dilemma game, whereas they play a 'stag-hunt' game (Harsanyi and Selten 1998) at the medium reliability level.

In Figure 2.16, we present the payoffs for C and S when they choose CDSS or SSS with different reliability levels of supplier U. We set $a = 100$, $c_1 = 5$, $c_2 = c_3 = 10$ and vary $\alpha \in \{0.3, 0.5, 0.8\}$. As in section 2.6.1, the profit of the firms with CDSS is higher than the profit with SSS. So in equilibrium, both firms choose CDSS when α is high and choose SSS, by way of the the prisoner?s dilemma, when α is low. However, when the reliability level is medium, we see that there are two equilibria in pure strategies like in a stag-hunt game. If both firms collude to choose CDSS before the game, the payoffs are $(951.4, 951.4)$ when $\alpha = 0.5$, which leads to a Pareto dominant equilibrium. Otherwise they both fall prey to the prisoner's dilemma.

	Buyer S		
	Expected Profit	CDSS	SSS
Buyer C	CDSS	**982.2, 982.2**	1002.3, 849.0
	0.3 SSS	849.0, 1002.3	900.0, 900.0

Both firms choose CDSS when reliability is high with $\alpha = 0.8$.

	Buyer S		
	Expected Profit	CDSS	SSS
Buyer C	CDSS	**951.4, 951.4**	828.1, 918.8
	SSS	918.8, 828.1	**900.0, 900.0**

Two equilibria in pure strategies where both buyers choose either CDSS or SSS, $\alpha = 0.5$.

	Buyer S		
	Expected Profit	CDSS	SSS
Buyer C	CDSS	930.8, 930.8	704.1, 962.4
	SSS	962.4, 704.1	**900.0, 900.0**

Both firms choose SSS when reliability is low with $\alpha = 0.3$. This game is the classical 'prisoner's dilemma'.

Figure 2.16. Equilibrium sourcing strategies for buyers with different reliability of supplier U. The payoffs for Buyers C and S are represented by Π_C, Π_S.

2.6.3 Capacity Reservation by Firm C

In some cases, a contingent dual sourcing firm may reserve capacity with a reliable supplier to mitigate disruption. Here we study the impact of capacity reservation by Firm C with Supplier R on the equilibrium order quantities and the profits of the firms. Let C pay c_r to Supplier R for each unit of reserved capacity. This fee is charged as an insurance against a possible disruption. R charges c_3^r for each unit purchased after disruption. Clearly, if $c_3^r = c_3$, then no capacity will be reserved and the entire analysis in section 2.3 holds. To avoid trivial case in this section , we will assume that $c_r > 0$ and $c_3^r < c_3$. One would expect that the firm may reserve capacity if the reservation cost is not too high and/or the difference $c_3 - c_3^r$ is not too small. For our analysis therefore, we will fix c_3^r and obtain a threshold T_r such that the firm will not reserve capacity if $c_r > T_r$.

Proposition 10. *Whenever a firm chooses to reserve capacity, the amount reserved will be equal to the emergency order quantity. Moreover, the capacity reserved decreases in c_3^r and remains unchanged in c_r.*

We next analyze the cases presented in section 2.3, now with Firm C having an option to reserve capacity with Supplier R.

Case 1 with Capacity Reservation

The sequence of events in Case 1 is shown in Figure 2.17.

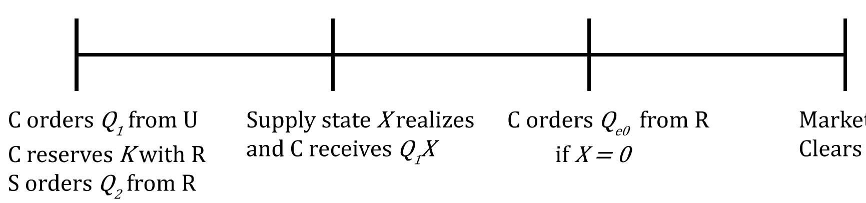


Figure 2.17. Sequence of events for Case 1 with capacity reservation.

In words, C and S simultaneously order Q_1 and Q_2 from Suppliers U and R, respectively, and C reserves a capacity K with Supplier R at the same time it orders from Supplier U. Then the supply state X realizes, C receives the quantity $Q_1 X$ from Supplier U, and S receives the quantity Q_2 from Supplier R. In the next stage, C places an emergency order Q_{e0} if $X = 0$. Then the market clears and the profits of C and S are realized.

What is played is a multi-stage game in which C places an emergency order from Supplier R upon the realization of the supply state X, whereas, before this realization, both C and S have already ordered Q_1 and Q_2 simultaneously from Suppliers U and R, respectively, and C has reserved capacity K from Supplier R. We use backward induction to obtain the equilibrium solution. That is, C's emergency order quantity response will be given as a feedback function $q_e(Q_1, Q_2, x)$, where x is the realization of X. If Supplier U does not default, i.e., $x = 1$, then clearly $q_e(Q_1, Q_2, 1) = 0$. However, when $x = 0$, C will maximize his profit to obtain $q_e(Q_1, Q_2, 0)$, i.e., $\max_{q_e}[(a - Q_2 - q_e - c_3^r)\, q_e]$. By solving this, we obtain the best response of buyer C given Q_1 and Q_2 as $q_e(Q_1, Q_2, 0) = \dfrac{(a - Q_2 - c_3^r)}{2}$. Thus, the entire feedback policy is

$$q_e(Q_1, Q_2, x) = \begin{cases} Q_{e0} = \dfrac{a - Q_2 - c_3^r}{2} & \text{if } x = 0, \\ Q_{e1} = 0 & \text{if } x = 1. \end{cases} \tag{2.38}$$

Next we solve the Nash game between C and S, knowing C's emergency order quantity reaction function. That is, C and S obtain Q_1, K and Q_2 simultaneously by maximizing their respective expected profits. In view of (2.38), therefore, we have the following simultaneous maximization problems:

$$\max_{Q_1, K} \quad \left[\alpha\,(a - Q_1 - Q_2 - c_1)\, Q_1 - c_r K + (1 - \alpha)\left(\frac{a - Q_2 - c_3^r}{2}\right)^2\right], \tag{2.39}$$

$$\max_{Q_2} \quad \left[\alpha\,(a - Q_1 - Q_2 - c_2)\, Q_2 + (1 - \alpha)\left(a - \frac{a - Q_2 - c_3^r}{2} - Q_2 - c_2\right) Q_2\right] \tag{2.40}$$

Solving the first-order condition with $K \geq Q_{e0}$ gives

$$K^{1*} = Q_{e0} \ , \ Q_1^{1*} = \frac{(1+\alpha)(a-2c_1)+2c_2-(1-\alpha)c_3^r}{2\,(\alpha+2)} \text{ and } Q_2^{1*} = \frac{a+\alpha c_1-2c_2+(1-\alpha)c_3^r}{\alpha+2}. \tag{2.41}$$

These are indeed the equilibrium order quantities since the objective functions (2.39) and (2.40) are jointly strictly concave in K, Q_1 and Q_2. The equilibrium can now be expressed as the triple $(Q_1^{1*}, Q_2^{1*}, Q_e^{1*})$, where Q_e^{1*} is the random variable $Q_e^{1*} = q_e(Q_1^{1*}, Q_2^{1*}, X)$. Inserting $Q_e^{1*} = 0$ when $X = 1$ and $K^{1*} = Q_{e0} = \dfrac{(1+\alpha)a-\alpha c_1+2c_2-3c_3^r}{2\,(\alpha+2)}$ into the objective functions (2.39) and (2.40), we obtain the equilibrium expected profits for C and S, respectively, as

$$\begin{aligned} E\left(\Pi_C^1\right) &= \frac{1}{4\,(\alpha+2)^2}\Bigg[\alpha\left[(1+\alpha)(a-2c_1)+2c_2-(1-\alpha)c_3^r\right]^2 \\ &\qquad +(1-\alpha)\left[(1+\alpha)a-\alpha c_1+2c_2-3c_3^r\right]^2 \\ &\qquad -2(\alpha+2)c_r\left[(1+\alpha)a-\alpha c_1-2c_2-3c_3^r\right]\Bigg], \\ E\left(\Pi_S^1\right) &= \frac{(1+\alpha)\left(a+\alpha c_1-2c_2+(1-\alpha)c_3^r\right)^2}{2(\alpha+2)^2}. \end{aligned}$$

The expected total market output is

$$E\left(S^1\right) = \alpha\left(Q_1^{1*}+Q_2^{1*}\right)+(1-\alpha)\left(Q_e^{1*}+Q_2^{1*}\right) = \frac{(3+\alpha)a-\alpha(1+\alpha)c_1-2c_2-(1-\alpha^2)c_3^r}{2\,(\alpha+2)}.$$

By comparing the profit of firm C in section 2.3.1 with the profit above, we find that the firm will reserve capacity with Supplier R when

$$\begin{aligned} c_r < \ T_r = \frac{1}{2(\alpha+2)(\alpha a+a-\alpha c_1-2c_2-3c_3^r)}\Big\{&-\alpha(\alpha(a-2c_1+c_3)+a-2(c_1+c_2)-c_3)^2 \\ &+(\alpha-1)(\alpha a+a-\alpha c_1+2c_2-3c_3)^2-(\alpha-1)(\alpha a+a-\alpha c_1+2c_2-3c_3^r)^2 \\ &+\alpha(a(\alpha+1)-2(\alpha+1)c_1+2c_2+(\alpha-1)c_3^r)^2\Big\}, \end{aligned}$$

and Firm C will not reserve capacity with Supplier R when c_r exceeds T_r.

Other Cases with Capacity Reservation

Analyses of the other cases are similar to the analysis for Case 1 above. We can derive T_r in each case, and if c_r is below the corresponding threshold, then the quantities ordered by firms C and S can be obtained simply by replacing c_3 with c_3^r in the formulas obtained in each case. The profit of firm C in each case is obtained by replacing c_3 with c_3^r in the derived formula and then subtracting the cost $c_r Q_{e0}$ of reserving the capacity $K = Q_{e0}$ at a cost c_r from the profit. We now summarize the threshold T_r for Cases 2-7.

Case 2:

$$\begin{aligned} T_r = & \frac{1}{2(a(\alpha+1)(\alpha+2)+\alpha(-2(\alpha+1)c_1+2c_2+(\alpha-3)c_3^r)+4c_2-6c_3^r)} \\ & \Big\{-2\alpha^3\left(a^2-a(3c_1-2c_2+c_3)+2c_1^2+c_1(5c_3-6c_2)+4c_3(c_2-c_3)\right) \\ & \left(a^2+a(6c_3-8c_1)+8c_1^2-4c_1(c_2+c_3)+c_3(4c_2-3c_3)\right) \\ & +2\alpha^3\left(a^2-a(3c_1-2c_2+c_3^r)+2c_1^2+c_1(5c_3^r-6c_2)+4c_3^r(c_2-c_3^r)\right) \\ & \left(a^2+a(6c_3^r-8c_1)+8c_1^2-4c_1(c_2+c_3^r)+c_3^r(4c_2-3c_3^r)\right) \\ & +2\alpha^3(c_1-c_3)(a-2c_1+c_3)+2\alpha^3(c_3^r-c_1)(a-2c_1+c_3^r)-(a+2c_2-3c_3)^2 \\ & +(a+2c_2-3c_3^r)^2\Big\}. \end{aligned}$$

Cases 3 and 5: $T_r = \dfrac{(1-\alpha)(c_3-c_3^r)(a+c_2-c_3-c_3^r)}{a+c_2-2c_3^r}$.

Case 4: $T_r = \dfrac{4(1-\alpha)(c_3-c_3^r)(a+c_2-c_3-c_3^r)}{3(a+c_2-2c_3^r)}$.

Cases 6 and 7: $T_r = \dfrac{(1-\alpha)(c_3-c_3^r)(6a-10\alpha c_1+12c_2+(5\alpha-9)(c_3-c_3^r))}{4(a-\alpha c_1+2c_2+(\alpha-3)c_3^r)}$.

2.7. Concluding Remarks

We have presented and analyzed a framework to study contingent dual sourcing strategy (CDSS) and sole sourcing strategy (SSS) under competition and supply disruption. We find that the realized supply state of an unreliable supplier and the competitor's time to place an order are critical to the profits of a buyer that operates under supply disruption. Since a buyer cannot control the supply disruption, we propose that he orders at a strategic time that effectively mitigates the negative effects of the supply disruption on his profit. Various managerial insights from the analysis, profit comparisons, computations and study of extensions are summarized below:

- Even though CDSS has a cost advantage over SSS, it does not necessarily dominate SSS. The cost advantage of CDSS depends also on how reliable the cheaper, unreliable supplier is. That is, when his reliability level is high, the cost advantage can be significant, making CDSS a better approach. On the other hand, when the reliability is low, SSS can be a superior strategy.

- It is interesting as well as surprising that for a firm using either sourcing strategy, the maximum profit is in the case when he places the order before his competitor (who adopts the other strategy) does.

- Conventional sourcing predicts that the expected total market output in a monopoly should increase with the reliability level of the supplier. However, there is a scenario (Case 1) in which the expected total competitive market output decreases as the reliability level of the supplier increases.

- In equilibrium, asymmetric firms with different sourcing costs may choose different sourcing strategies depending on the reliability level of the unreliable supplier.

- In equilibrium, symmetric firms choose the same sourcing strategy. Specifically, when the reliability of the unreliable supplier is high and his costs are sufficiently low, the firms choose CDSS, otherwise they choose SSS.

- When CDSS is adopted with a capacity reservation with the reliable supplier, the equilibrium capacity to reserve is equal to his emergency order quantity.

- With capacity reservation, we derive the thresholds for per unit capacity reservation cost above which the CDSS firm does not reserve capacity.

There are possible future extensions of our research that are worth considering. One is a study of the competitive buying behavior with CDSS and SSS when suppliers have capacity limits. As a result, the buyers may not be able to order up to the level that they could without the capacity limits. This would mean that having the suppliers with limited capacities will have implications on the strategy of the buyers, and these will be worth examining. A more detailed study of endogenous sourcing than that carried out in section 2.6.1 would reveal how the cost asymmetry and the reliability level of the unreliable suppliers interact. Specifically, what are the threshold level of the unreliability for the choice of different strategies given the sourcing costs.

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CHAPTER 3

PAY-PER-UNIT VS. SUBSCRIPTION PRICING FOR EXPERIENTIAL PRODUCTS UNDER COMPETITION

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3.1. Introduction

Pricing modality describes the "rules of the game" in a given market, i.e., it defines the interaction among buyers, sellers, and intermediaries in a market to determine the prices for a transaction (Özer and Philips 2012). Examples of pricing modalities include list pricing, customized pricing, pay-per-unit pricing, subscription pricing, and two-part tariff, etc. Distributors (sellers) offering identical products in a market can employ different pricing modalities to create different values for consumers (Choudhary 2010). What pricing modalities should these competing distributors employ to attract consumers to purchase experiential products, when they themselves have a purchase contract with a common content provider?

Consumer's usage (consumption level) and their choice of a distributor for the product depends on the pricing modality (e.g., pay-per-unit, subscription) and the price. Consumers who choose subscription pricing, have the option to experience a set of products offered by the distributor. Whereas, consumers who choose pay-per-unit have a restricted usage of the product due to money constraint. We categorically study the relevance of two ubiquitous pricing modalities for experiential products: *pay-per-unit* and *subscription* pricing, when distributors source products from a common content provider with either *licensing* or *wholesale price* contract.

"Experiential products are those products which consumers choose, buy and use solely to experience and enjoy." (Cooper-Martin 1991). The primary difference between experiential products, and other products and services arises due to expenditure of time by the consumers. We focus our attention to the study of digitized experiential products such as movies, music and electronic books which have zero marginal cost of production and requires consumers to expend time along with money. In experiential products industries, such as video streaming industry, subscription distributors, such as Netflix, often have a B2B licensing contract and the pay-per-unit distributors, such as iTunes, have a B2B wholesale price contract with the content provider (Movie Studio). The licensing contract is a fee charged by the content provider for providing the product to the distributor. This fee is independent of the distributors sales. For example, Netflix pays a fixed fee to license its content from Disney and Starz (see, for example, the Form10-K of Netflix). The wholesale price contract for experiential products is different from those contracts designed for physical product distribution. In particular, the wholesale price is paid after the realization of the consumer demand in case of experiential products. For example, distributors such as iTunes, and Barnes & Noble operate under a wholesale price contract with movie and books providers and pay to the movie studio after sales to customers (see, for example, the Nook website. Nook is the e-book reader sold by Barnes & Noble).

We answer the following questions in this chapter: How do consumers choose between different pricing modalities used to sell identical experiential products? When a vertically integrated monopolist distributor offers experiential products to such consumers, which pricing modality gives her a higher profit? What are the equilibrium prices in the market when the pay-per-unit pricing distributor and the subscription pricing distributor compete for consumers? What are the effects on this equilibrium due to the B2B contract with a common content provider? Is there any stable price equilibrium? Finally, what should be the first best pricing choice for a distributor who enters the experiential products market in the presence of an incumbent distributor with a given pricing modality?

We propose a general discrete choice model for experiential products with pay-per-unit and/or subscription pricing from first principles of utility maximization problem (UMP) of a rational consumer. Then we use this model to analyze the prices, profits, and market penetrations of the distributors under monopoly and under competition.

Literature Review

Pay-per-unit and subscription pricing in competitive markets has been studied in different contexts such as – (a) without capacity constraints (Fishburn and Odlyzko 1999); (b) with capacity constraints (Essegaier et al. 2002, Bitran et al. 2008); (c) limit on usage (Randhawa and Kumar 2008); (d) service constraints (Cachon and Feldman 2011); and (e) positive network externality (Oh et al. 2012). These works aim to find a sustainable business model for distributors under competing subscription and pay-per-unit pricing modalities.

Fishburn and Odlyzko (1999) study competition between two distributors that offer identical electronic (or online) goods (or services) with pay-per-unit and subscription pricing. They assume that distributors have negligible marginal costs and maximize revenues from consumers who are cost minimizers and vary on their usage (assumed exogenously given with a pdf on usage rate independent of prices). They show that this type of pricing-modality based service differentiation often leads to price wars and stable prices do not occur without collusion. They show that even when the consumers are willing to pay a fixed-subscription premium or have a budget constraint the distributors end up in a price war situation. However, in special cases such as pre-announcement of prices by a distributor, cooperation, and covert collusion, they show the existence of a stable pricing equilibria. We also consider competing profit maximizing distributors sourcing from a common content provider under different B2B contracts – wholesale price contract and licensing contract. In this chapter, we show that when the pay-per-unit distributor and the content provider have a zero wholesale price (no marginal costs), the distributors end up in a price war. Therefore, the contract structure with marginal costs not only results in stable equilibrium prices but also enriches the study of subscription vs. pay-per-unit pricing. Essegaier et al. (2002) study monopoly and competitive pricing for capacity constrained access service distributors with subscription, pay-per-unit, and two-part tariff pricing modalities. They show that the choice of modality is governed by consumer usage heterogeneity and the service capacity of the distributor. They assume that consumer usage is inelastic to the changes in price and independent of the modality. We assume that the consumer usage depends on the price and the pricing modality as shown by Altmann and Chu (2001), specifically we show that a consumer's usage is higher with subscription as compared to pay-per-unit. In absence of capacity constraints two-part tariff are always optimal as pay-per-unit and subscription pricing are its special cases. Essegaier et al. (2002) show that pay-per-unit pricing is optimal only when the industry has sufficiently small capacity thus resulting in an uncovered market and subscription pricing is optimal when there is sufficient excess capacity. We show that for a monopolist in a market of highly heterogenous consumer valuations subscription pricing is optimal with an uncovered market, i.e., maximizing market penetration is not the guiding principle for the distributor. This result is driven by our consumer choice framework where consumer usage is elastic in the subscription and pay-per-unit price. We model consumer utility explicitly, so that the consumer usage is an outcome of the market prices, pricing modalities, and the consumer preferences, rather than exogenously given. We also model the upstream interaction of distributors with a common content provider through wholesale and licensing contracts.

Randhawa and Kumar (2008) compare pay-per-unit pricing with subscription pricing; where subscription pricing is employed with a limit on concurrent rentals or usage. They compare social welfare and consumer surplus under the two pricing modalities and can be higher in the subscription option. We also derive and compare consumer surplus and social welfare in the study. Bitran et al. (2008) study the effect of a service provider's policies on pricing and service level on the size of its consumer base and profitability. Our work differs from them in several ways. We assume that all consumers can be served parallel and distributors are not facing any capacity constraints. We do not study the effect of distributor's capacity and service level constraints on consumer usage and the equilibrium prices. In the context of experiential products such as online streaming of movies, music and e-books distributors can typically provide full service to all the consumers in parallel, without any constraints on capacity and service. Therefore, in this chapter we primarily focus on the impact of a pricing modality on consumer behavior, service providers and content provider.

Subscription pricing vs. pay-per-unit has been studied by Cachon and Feldman (2011) in the context of congestions in service. They study monopoly pricing without marginal costs in a market of homogenous consumers. Consumer homogeneity controls for the advantage of subscription pricing due to segmentation. To model the consumers in such a setting, they assume consumers arriving in queues to the market gain a utility less the cost from the service per unit time. They assume that the consumer's usage depends on the congestion in the system (negative externality) and does not depend on the pricing scheme. They show that subscription pricing is more effective even when consumers dislike congestion. In our chapter, we assume that distributors are able to deliver service to all the consumers without any congestion such as online streaming of movies by Netflix and iTunes, etc.

Oh et al. 2012 study a game theoretic model of customer choice between subscription and pay-per-unit pricing, where consumers benefit from the size of the service network, i.e., positive network externality without capacity constraints. For e.g., voice calling consumers have a higher usage if their friends are on the network and their decision depends on their friend's usage and the choice of voice service provider such as AT&T, Verizon and Sprint etc. in the United States. They derive equilibrium consumer usage given the prices charged by a monopolist who offers both subscription and pay-per-unit prices. They do not study price optimization by the distributor or competition between the distributors in their work. In this chapter, we do not study the positive/negative effects of network size on consumer usage and channel choice.

In section 3.2, we develop a model that captures the behavior of consumers when they confront pay-per-unit and subscription pricing modalities for experiential products, that includes time and money as costs to the consumer. In section 3.3, we study a vertically integrated monopolist who sells experiential product with subscription and/or pay-per-unit pricing to find her optimal pricing modality. In section 3.4, we study the interaction between two competing distributors that operate in a market with pay-per-unit and subscription pricing, respectively. Section 3.5 discusses the optimal pricing modality for an entrant in the presence of an incumbent distributor with a specified pricing modality. Finally, we discuss our findings in this setting and propose possible future research directions in section 3.6.

3.2. Consumer Choice for Experiential Products

We model consumers' utility for experiential products as a sum of their valuation of the product less the non-pecuniary cost of using the associated experiential product and the price charged by distributor. Their channel-choice and product usage are therefore an outcome of this general discrete choice model.

A consumer gains a utility $u(k)$ and incurs a time cost $t(k)$ by consuming experiential product $k \in \mathcal{M}$, where $\mathcal{M}$ is her consideration set. We assume that $u(k)$ and $t(k)$ are independent of the distributor providing k. This means that consumers can differentiate the product only due to difference in price across distributors. This is particularly valid for online streaming services where the enjoyment of watching movies does not depend whether consumers choose iTunes or Netflix. The experience of using a health club on a particular visit is the same whether a consumer pay for each visit or she choose to buy membership (subscription). In line with the neoclassical consumer choice theory, our consumers are rational who decide the usage of the product, and the distributor that maximizes their overall surplus based on the utility, time cost and the monetary payment to distributor for the associated usage. This leads to self-selection of distributors by consumers.

Given consumer's usage set $\mathcal{M}$, her surplus is the additive utility obtained from the usage $\sum_{k \in \mathcal{M}} u(k)$ less the time cost $\sum_{k \in \mathcal{M}} t(k)$, and the pecuniary cost p_{sub} or $p_{ppu}|\mathcal{M}|$ for subscription or pay-per-unit pricing, respectively.[1] A consumer's utility maximization problem (UMP) yields her usage sets $\mathcal{S}_{sub}$ and $\mathcal{S}_{ppu}$ from subscription or pay-per-unit pricing, respectively. We assume subscription distributor and pay-per-unit distributor provide same set of products $\mathcal{M}$. We define the net utility as $z(k) := u(k) - t(k)$. Therefore, if a consumer j chooses a set of $\mathcal{S}_{sub} \in \mathcal{M}$ and $\mathcal{S}_{ppu} \in \mathcal{M}$ products then she gets a total net utility of $\sum_{k \in \mathcal{S}_{ppu} \bigcup S_{sub}} z(k)$. Consumers choose channel and products that maximize their surplus.

[1]Where useful, we use *ppu* and *sub* subscripts to signify notation associated with the pay-per-unit and subscription pricing, respectively.

Without loss of generality, we assume $z_k \geq 0$, as consumers are rational and they can always choose not consume and get zero utility. Therefore, consumer solves the following problem to decide her usage set from both types of distributors

$$\max_{\{Sppu \in \mathcal{M}, \mathcal{S}_{sub} \in \mathcal{M}\}} \sum_{k \in \mathcal{S}ppu \bigcup \mathcal{S}_{sub}} z(k) - \mathbb{I}_{\mathcal{S}_{sub} \neq \phi} p_{sub} - |\mathcal{S}_{ppu}| p_{ppu}, \tag{3.1}$$

and $(\mathcal{S}^*_{sub}, \mathcal{S}^*_{ppu})$ is the optimal solution to (3.1). When oly subscription or only pay-per-unit pricing channels are available, the UMP for each consumer is:

$$\max_{\{\mathcal{S}_{sub} \in \mathcal{M}\}} \sum_{k \in \mathcal{S}_{sub}} z(k) - \mathbb{I}_{\mathcal{S}_{sub} \neq \phi} p_{sub}, \tag{3.2}$$

$$\max_{\{Sppu \in \mathcal{M}\}} \sum_{k \in \mathcal{S}_{ppu}} z(k) - |\mathcal{S}_{ppu}| p_{ppu}. \tag{3.3}$$

Further, let $\hat{\mathcal{S}}^*_{sub}$ and $\hat{\mathcal{S}}^*_{ppu}$ be the optimal solutions of (3.2) and (3.3), respectively.

3.2.1 General Properties

The general UMP model yields properties such as a unique channel choice of a consumer, and that the optimal usage set of a consumer can be always described as the first k products with highest net utilities, for some value of k. Therefore the UMP is transformed from an optimization problem over sets into an optimization problem over the number of products consumed.

The choice of a consumer for experiential products given the prices in the market is given by (3.1) – (3.3), which gives the channel choice as well as the optimal set of products for each consumer. Distributors offer a large array of products and the set of items considered by each consumer increases in the number of offerings. We show that when consumers have an order on the valuation of products such as ranking of movies, music or books then the consumption set (n-dimensional) of each consumer is identifiable by a number. Basically, if the consumer is experiencing $\mathcal{X} \subset \mathcal{M}$ set of products, it is the first $|\mathcal{X}|$ products in the available *ordered* set $\mathcal{M}$. Without loss of generality, the order of products is defined on a set of products $\mathcal{M}$ as: $z(j) \geq z(k)$ for $j < k \in \mathcal{M}$, i.e., the experiential utilities are non-increasing in the index.

Proposition 11. *a)* **Unique Channel Choice:** *Consumers choose either subscription or pay-per-unit distributor, or nothing but not both.*

b) **Consumer Choice in the Order of Valuation:** *If $i \in \mathcal{S}^*_{ppu}$, then $j \in \mathcal{S}^*_{ppu}$ for all $j \leq i \in \mathcal{M}$.*

c) **Consumers Increase their Usage with Subscription Pricing:** *When consumers choose subscription pricing over pay-per-unit pricing then they consume no less than their consumption under pay-per-unit pricing.*

Consumers experience products based on their ranking or order of the products. For example, consumers watch a movie they rank higher before their lower ranked movie. Netflix has a 'My List' option for every user which is the list of movies she wants to watch. Proposition 11 does not depend on the pricing scheme and extends to the setting for subscription pricing. It follows from Proposition 11 that consumers choose $m^*_{ppu} := \max\{i : i \in \mathcal{S}^*_{ppu}\}$ and $m^*_{sub} := \max\{i : i \in \mathcal{S}^*_{sub}\}$, the number of products from subscription and pay-per-unit distributors, respectively. Therefore, consumer choice set is identifiable by a number which helps simplify the analysis. Using Proposition 11, we can simplify (3.1) as:

$$\max\left\{\max_{0\leq m_{ppu}\leq|\mathcal{M}|}\sum_{i=1}^{m_{ppu}} z(k) - m_{ppu}p_{ppu}, \max_{0\leq m_{ppu}\leq|\mathcal{M}|}\sum_{i=1}^{m_{sub}} z(k) - m_{sub}\right\}. \tag{3.4}$$

Similarly, we can simplify (3.2) and (3.3) in terms of the number of products consumed. We know that every additional product consumed has less value than the previous product, i.e., $z(k)$ is non-increasing in k. We assume that the time cost $t(k)$ of a consumer increases in k, as consumers have limited time and the opportunity cost of time increases with shortage of time. Therefore, we expect $z(k)$ to be negative at a certain usage, i.e., where the consumers do not benefit from consumption even when the product is free. Therefore, $z(k)$ is a decreasing function in k and eventually becomes negative. Therefore, $U(x) := \sum_{i=1}^{x} z(i)$, the experiential utility is a concave function in x (decreasing differences, as $z(k)$ is decreasing) and attains a maximum level.

Under a general utilitarian framework for experiential products, Proposition 11 c) shows that there is a difference in consumer usage due to the pricing modality. Moreover, this difference is biased towards a higher usage under subscription pricing. The economic intuition behind this increased usage is the zero marginal pecuniary cost of consumption under subscription pricing, however consumers incur a positive marginal cost of time spent to experience the product. Proposition 11 is in confirmation with the findings of Altmann and Chu (2001) who have empirical validated this bias in consumer usage behavior.

3.2.2 An Analytical Model of Consumer Choice

In order to obtain insights about the behavior of consumers, distributors and the content provider in the market, we use an analytical model, which is a continuous approximation of the general UMP model with the optimization over the number of products consumed. Let $x \in \mathbb{R}^+$ be the quantity of products consumed. The form of the utility function $U(x)$ is approximated as $ax^b - cx$, where the parameters $a > 0$, $b \in (0,1)$ and $c > 0$ describe a consumer's general level of interest (or valuation) in the experiential products, sensitivity to choice and time cost, respectively.

Now we derive a consumer's usage levels, choice criterion, and surplus for given prices p_{sub} and p_{ppu} for markets which have – (a) subscription pricing only; (b) pay-per-unit pricing only; and (c) both subscription and pay-per-unit pricing.

Consumer Choice under Subscription Pricing

We use the analytical model and derive consumers' optimal usage given the subscription price, when only a subscription distributor is available. We observe that there is a segment of consumers with valuation above a threshold who purchase subscription, i.e., market penetration is not necessarily 100%. Their usages vary as a function of their valuation.

We consider a market with only subscription pricing and the price as p_{sub}. Consumers experience x unit of products when they choose subscription such that $ax^b - cx - p_{sub} \geq 0$, i.e., they have a positive experiential utility. The optimal usage level x^*_{sub} for such consumers is obtained by solving their UMP:

$$\max_{\{x \geq 0\}} \left[ax^b - cx - p_{sub}\right].$$

Then optimal usage level is given by

$$x^*_{sub} = \begin{cases} \left(\frac{ab}{c}\right)^{\frac{1}{1-b}} & \text{if } p_{sub} \leq \frac{c(1-b)}{b}\left(\frac{ab}{c}\right)^{\frac{1}{1-b}}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.5}$$

The condition in (3.5) gives an upper bound on the subscription price. If the price is higher than this bound, then the consumer does not subscribe.

From the above discussion, we see that subscription channel becomes increasingly attractive to consumers with a high valuation (high a) and a low time cost (low c). The usage x^*_{sub} is increasing in a and b, and decreasing in c. Therefore, consumers with a 'heavy' usage find subscription attractive. In Figure 3.1 consumers with a high a purchase subscription. Therefore, the market penetration for subscription is not necessarily 100%.

Consumer Choice under Pay-per-unit Pricing

We use the analytical model to derive consumers' optimal usage given the pay-per-unit price, when only a pay-per-unit distributor is available. Given the price charged by the pay-per-unit distributors p_{ppu}, a consumer solves

$$\max_{\{x \geq 0\}} \left[ax^b - cx - p_{ppu}x\right].$$

This gives the optimal usage of the consumer as

$$x^*_{ppu} = \left(\frac{ab}{c + p_{ppu}}\right)^{\frac{1}{1-b}}. \tag{3.6}$$

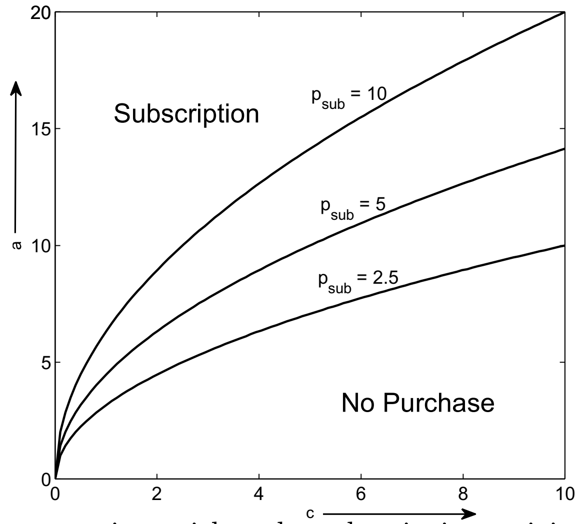


Figure 3.1. Market segmentation with only subscription pricing. Consumers with high general level of interest (a) and low time cost (c) purchase subscriptions when the selectivity is fixed at $b = 1/2$. *Market share decreases in the subscription price.*

Therefore, from (3.5) and (3.6) we see that the consumer who purchases experiential products via subscription consumes more than what she would consume with pay-per-unit pricing.

All consumers purchase from the pay-per-unit pricing distributor, i.e., the market penetration for pay-per-unit distributor is 100%. Consumers' usage vary as a function of their valuation and there may be consumers with very low valuations, who purchase infinitesimally small amounts of the product, and therefore pay low pay-per-unit fees. This is in contrast to subscription pricing, where in order to justify the subscription fee, a consumer needs to have a sufficiently high valuation and consume a sufficiently high number of products.

Consumer Choice when both Subscription and Pay-per-unit Pricing are available

We use the analytical model to derive consumers optimal usage given both subscription and pay-per-unit prices. We observe that there is a segment of consumers with valuation above a threshold, who purchase subscription. All others use the pay-per-unit option.

Given the prices p_{sub} and p_{ppu}, consumers choose the channel which gives them a higher surplus. They solve the problem as given in (3.4), which is given by

$$\max\left\{\max_{\{x\geq 0\}}\left[ax^b - cx - p_{sub}\right], \max_{\{x\geq 0\}}\left[ax^b - cx - p_{ppu}x\right]\right\},$$

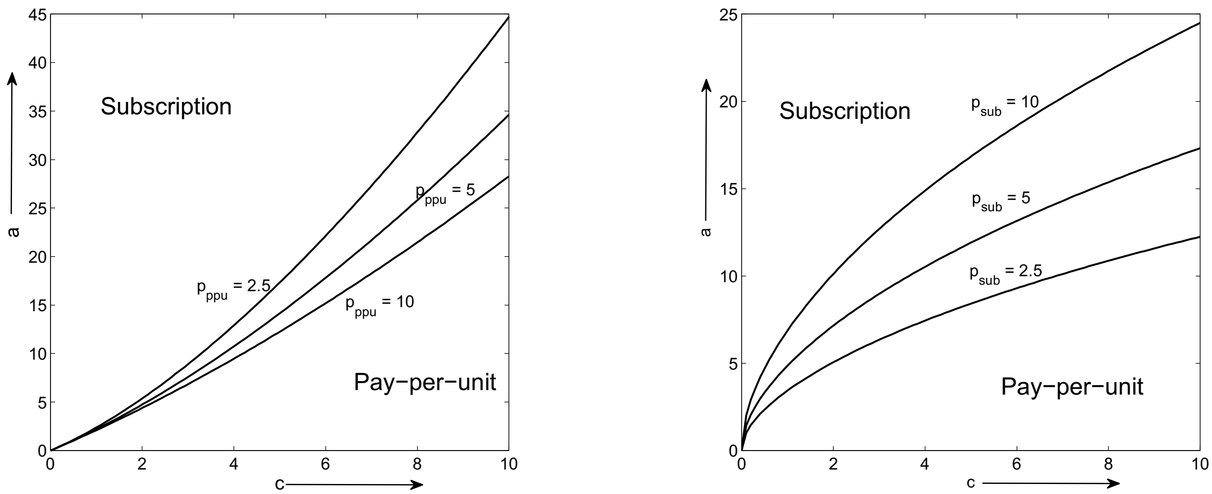
and we know that consumers choose subscription and pay-per-unit channels with usage x^*_{sub} and x^*_{ppu}, respectively. From (3.5) and (3.6), we obtain the condition for a consumer to choose subscription instead of pay-per-unit:

$$\frac{c(1-b)}{b}\left(\frac{ab}{c}\right)^{\frac{1}{1-b}} - p_{sub} \;\; \geq \;\; \frac{(1-b)(c+p_{ppu})}{b}\left(\frac{ab}{c+p_{ppu}}\right)^{\frac{1}{1-b}} \tag{3.7}$$

Again, subscription channel becomes increasingly attractive to consumers with a high valuation (high a), low selectivity (high b) and a low time cost (low c). Therefore, consumers with a 'heavy' usage find subscription attractive to pay-per-unit pricing which is more attractive to consumers with 'light' usage. The two market segments are shown in Figures 3.2(a) and 3.2(b) which gives the channel choice with varying subscription and pay-per-unit price, respectively. Figure 3.2(a) is obtained by varying subscription price $p_{sub} \in [0, 10] := c$ and $b = 1/2$. Figure 3.2(b) is obtained by varying pay-per-unit price $p_{ppu} \in [0, 10] = 1.6c + 4$ and $b = 1/2$. At given (p_{ppu}, p_{sub}) consumers above the indifference curve choose subscription and those below the curve choose pay-per-unit. We observe that as the subscription distributor reduces the price, consumers with a high time cost c and a high valuation a switch to subscription. From Figure 3.2(b) we see that the number of consumers switching to subscription is decreasing with change in pay-per-unit price reaching the maximum market share.

3.3. Vertically Integrated Monopoly Pricing of Experiential Products

In this section, we assume that the general level of interest parameter a is distributed uniformly between a_1 and a_2, with $0 \leq a_1 < a_2$ and that there are N consumers in the market. The selectivity parameter b and the time cost parameter c are assumed to be the same for all consumers. We use the analytical utility model to derive the optimal price for a monopolist content provider who is also a distributor. In other words, we study a vertically integrated monopolist selling experiential products.



(a) Varying subscription pricing.

(b) Varying pay-per-unit pricing.

Figure 3.2. Market segmentation in the presence of both pay-per-unit and subscription pricing.

Subscription Pricing

We obtain the optimal subscription price and profit for a monopolistic subscription distributor in closed form. We observe that the expressions for the subscription price, profit and market penetration depend on the level of consumer heterogeneity. Low heterogeneity is indicated as $a_1 > a_2/(2-b)$ and high heterogeneity is indicated as $a_1 \leq a_2/(2-b)$.

Proposition 12. *a) Low Heterogeneity ($a_1 > a_2/(2-b)$): The optimal subscription price is* $\frac{c(1-b)}{b}\left(\frac{a_1 b}{c}\right)^{\frac{1}{1-b}}$ *and the market penetration is 100%.*

b) High Heterogeneity ($a_1 \leq a_2/(2-b)$): The subscription price is $\frac{c(1-b)}{b}\left(\frac{a_2 b}{c(2-b)}\right)^{\frac{1}{1-b}}$ *and the market penetration is* $\frac{a_2(1-b)}{(a_2-a_1)(2-b)}$.

Most important result from Proposition 12 is that the market penetration is 100% when heterogeneity is low and less than 100% if the heterogeneity is high. The optimal subscription price depends only on a_2, as a subscription monopolist can set the highest price and still sell to a consumer with the interest level a_2. Decreasing the price will attract more consumers with valuation lower than a_2 but as long the decrease in price compensates for the gain in market penetration the distributor can decrease the price to sell to more consumers. At some price point there is a consumer with valuation $\hat{a}(p_{sub})$ who is a break-even consumer, i.e., she is the consumer with the lowest valuation who is indifferent between subscription and no purchase and marginally purchases subscription. Depending on the heterogeneity of consumers this break-even consumer can be real when the heterogeneity is high, i.e., $a_1 < \hat{a}(p_{sub}) < a_2$ or virtual when the heterogeneity is high $a_1 \geq \hat{a}(p_{sub})$. Therefore, when we observe high heterogeneity in the market the optimal subscription price depends only on a_2 but not on a_1.

When the monopolist chooses subscription pricing with price per consumer p_{sub}, we can derive the aggregate demand N_{sub}^{mon}, i.e., the number of subscribers. The attractiveness of subscription pricing increases with a, and there is a consumer whose interest is $\hat{a}(p_{sub})$, and this consumer is indifferent between not consuming anything and consumption with subscription pricing. This is also illustrated in Figure 3.3 and is given by

$$\hat{a}(p_{sub}) := \frac{c}{b}\left(\frac{bp_{sub}}{c(1-b)}\right)^{1-b}. \tag{3.8}$$

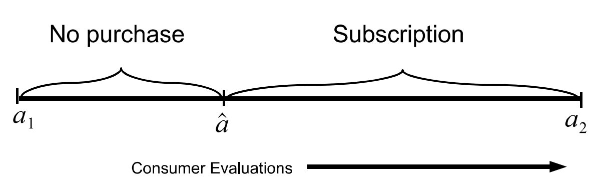


Figure 3.3. Market segmentation with subscription pricing. Consumers with high a purchase subscription.

The revenue depends on the number of subscribers for a monopolist subscription distributor, which can be written as follows

$$\begin{aligned} N_{sub}^{mon}(p_{sub}) &= N \int_{a_1}^{a_2} \frac{1}{a_2 - a_1} \mathbb{1}_{\{a \geq \hat{a}(p_{sub})\}} da \\ &= \begin{cases} N & \text{if } a_1 > \hat{a}(p_{sub}), \\ \dfrac{N\left(a_2 - \hat{a}(p_{sub})\right)}{a_2 - a_1} & \text{if } a_2 \geq \hat{a}(p_{sub}) \geq a_1, \\ 0 & \text{if } a_2 < \hat{a}(p_{sub}), \end{cases} \end{aligned}$$

Pay-per-unit Pricing

We obtain the optimal subscription price and profit for a monopolistic pay-per-unit distributor in closed form. Here, the forms of the expressions do not depend on whether heterogeneity is high or low. The optimal price turns out to have a very simple expression, which is $c(1-b)/b$, and is independent of a_1 and a_2.

Proposition 13. *The optimal pay-per-unit price is* $\dfrac{c(1-b)}{b}$ *and the market penetration is 100%.*

The independence of the optimal pay-per-unit price from a_1 and a_2 is an artefact of the specific form of the analytical model of consumer choice we use which is linear in c and proportional to a. A consumer with a low interest in the experiential product has a very low value of a compared to other consumers. This refers to those consumers who consume rarely, such as people who watch movies or read books only once in a while.

Both Subscription and Pay-per-unit Pricing are available

When a monopolist can sell experiential products using both subscription and pay-per-unit pricing modalities, it is optimal for him to operate through subscription pricing alone. This is not a straightforward result and specially when the heterogeneity is high the market penetration is not 100 %, and one could argue that the consumers which do not subscribe can be sold experiential products via pay-per-unit pricing resulting in additional revenues. However, counter to this intuition we prove that the monopolist maximizes the profit by choosing to sell experiential products through subscription pricing alone.

A monopolistic distributor will have higher profits using subscription pricing, as opposed to pay-per-unit pricing or both pay-per-unit and subscription pricing, regardless of the parameters $a_1,\ a_2,\ b = 1/2,\ c$ and N.

Proposition 14. *A monopolist selling experiential products should choose subscription pricing modality to maximize the revenue.*

Proposition 14 is obtained by comparing profits under subscription and pay-per-unit pricing. A vertically integrated monopolist distributor selling experiential products earns more with subscription pricing than pay-per-unit pricing, regardless of the consumer heterogeneity in the market. The lucrativeness of subscription pricing for experiential products categorically those which are digital in nature is due to no capacity limits and zero marginal costs. These are important factors that leads a monopolist to sell via subscription even though the consumers increase their usage, however it does not cost the monopolist to support this higher level of consumer usage. Therefore, he charges more to the consumers pertaining to this higher usage, thereby increasing profits.

3.4. Competitive Pricing of Experiential Products

Consider a single content provider selling its product to two distributors which are competing in the same market as shown in Figure 3.4. One of the distributors is using subscription pricing and has a licensing contract with licensing cost K with the content provider. The other distributor is using pay-per-unit pricing and has a wholesale price contract with the content provider. Under the wholesale price contract the pay-per-unit distributor pays the content provider w for every product sold to the consumer after the realization of consumer demand.

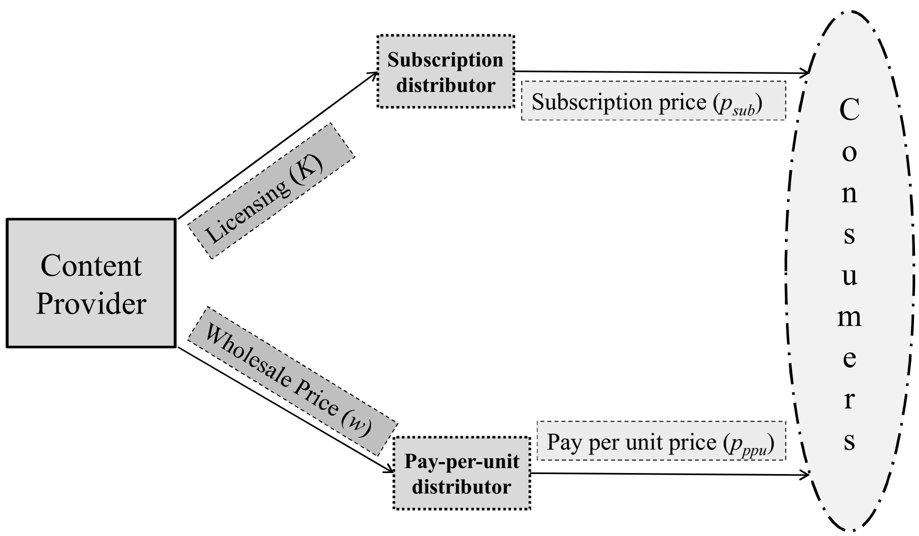


Figure 3.4. Experiential product distribution channel: contracts and pricing structure for pay-per-unit and subscription distributors.

In experiential products industries, such as video streaming industry, subscription distributors, such as Netflix, often have a B2B licensing contract and the pay-per-unit distributors, such as iTunes, have a B2B wholesale price contract with the content provider (Movie Studio). The licensing contract is a fee charged by the content provider for providing the product to the distributor. This fee is independent of the distributors sales. For example, Netflix pays a fixed fee to license its content from Disney and Starz (see, for example, the Form10-K of Netflix). The wholesale price contract for experiential products is different from those contracts designed for physical product distribution. In particular, the wholesale price is paid after the realization of the consumer demand in case of experiential products. For example, distributors such as iTunes, and Barnes & Noble operate under a wholesale price contract with movie and books providers and pay to the movie studio after sales to customers (see, for example, the Nook website. Nook is the e-book reader sold by Barnes

& Noble). We also discuss other combinations of contract types for the distributors towards the end of this section.

The game among the distributors and the content provider is often modeled as a two-stage game in which the parties have the same information about each other's business (Özer and Wei 2006). In the first stage, the terms of the wholesale price contract and the licensing contract are determined. Depending on the contractual power structure of the players, the content provider or the corresponding distributor may be setting the contract terms and we study all possible combinations later in the section. In the second stage, the distributors play a Nash pricing game. In this section, we assume that the selectivity parameter $b = 1/2$. We first study the pricing game between the distributors and then determine the equilibrium contracts between the content provider and the distributors.

3.4.1 Pricing Game between Distributors

This is the second stage of the game. Given the wholesale price $w \geq 0$ and licensing fee $K \geq 0$, we identify the unique equilibrium pay-per-unit and subscription prices in closed form.

The subscription demand is the total number of consumers that choose subscription pricing, while the demand faced by pay-per-unit distributor is the total units of product demanded by all consumers who do not choose subscription pricing. The attractiveness of subscription pricing is increasing in a, and there is a consumer with interest $\tilde{a}(p_{sub}, p_{ppu})$ who is indifferent between pay-per-unit and subscription pricing. Consumers with a valuation higher than the threshold $\tilde{a}(p_{sub}, p_{ppu})$ purchase subscription as shows in Figure 3.5. This is given by

$$\tilde{a}(p_{sub}, p_{ppu}) := \frac{1}{b} \left\{ \frac{b p_{sub}}{(1-b)\left[c^{\frac{-b}{1-b}} - (c + p_{ppu})^{\frac{-b}{1-b}}\right]} \right\}^{1-b}. \tag{3.9}$$

Note that, $\tilde{a}(p_{sub}, p_{ppu})$ is *increasing* in p_{sub} and *decreasing* in p_{ppu}.

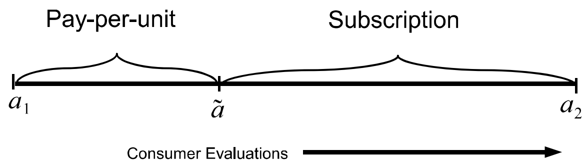


Figure 3.5. Market segmentation with subscription pricing and pay-per-unit. Consumers with high a purchase subscription.

The number of subscribers for the subscription pricing distributor and the demand for the pay-per-unit distributor as unit of products are:

$$N_{sub}(p_{sub}; p_{ppu}) \quad = \quad N \int_{a_1}^{a_2} \frac{1}{a_2 - a_1} \mathbb{I}_{\{a \geq \tilde{a}(p_{sub}, p_{ppu})\}} da,$$

and

$$D_{ppu}(p_{ppu}; p_{sub}) \quad = \quad N \int_{a_1}^{a_2} \frac{1}{a_2 - a_1} \mathbb{I}_{\{a < \tilde{a}(p_{sub}, p_{ppu})\}} \frac{a^2}{4(c + p_{ppu})^2} da$$

Theorem 5. *a) There is a unique price equilibrium, which can be calculated in closed form (see Appendix C) and increases in w but does not depend on K.*

b) If wholesale price is positive, the distributors coexist profitably in the equilibrium with positive prices and profits. If wholesale price is zero there is price wars, and the distributors make zero profits with equilibrium prices set to zero.

c) The equilibrium market penetration for the subscription distributor is identical to the monopolist subscription distributor, i.e., subscription distributor gets whole market with low heterogeneity of consumers and shares the market with pay-per-unit distributor with pay-per-unit distributor and gets market penetration of $\frac{a_2(1-b)}{(a_2 - a_1)(2-b)}$.

The wholesale price contract between the pay-per-unit and the content provider drives the equilibrium prices, and the licensing cost does not affect the equilibrium prices as it is a sunk cost. Theorem 5 c) is valid for general b but not for any model of consumer choice.

From Theorem 5 b) we see that when $w = 0$ the distributors engage in price wars. Fishburn and Odlyzko (1999) derived a similar result without considering an upstream player and its effects. They show that when the marginal costs to the distributors are zero, the distributors engage in a price war. On the other hand, we show that in the presence of an upstream content provider the variable cost to the pay-per-unit distributor is not zero and hence we have a stable equilibrium. Consideration of the wholesale price contract with a content provider leads to a strikingly different equilibrium which is stable and therefore it is important to carefully consider any upstream contracts of the distributor.

Equilibrium Demand and Profit: With the knowledge of equilibrium prices we now find the equilibrium market characteristics, i.e., the demands, profits and the market shares for the distributors.

The equilibrium market share for pay-per-unit distributor is zero, when the consumer heterogeneity is low ($2a_2 < 3a_1$) or the wholesale price is too high. It is again optimal for the subscription pricing distributor to adopt a market penetration strategy as in a monopoly. The subscription distributor has the same market penetration in equilibrium as a monopolist. When the content provider announces a very large wholesale price, it drives the pay-per-unit distributor out of competition and then the subscription distributor charges the monopoly price p_{sub}^{mon}.

3.4.2 Equilibrium Contracts Between the Content Provider and Distributors

The terms of the wholesale price contract can be set by either the content provider or the pay-per-unit distributor. Similarly, the terms of the licensing contract can be set by the content provider or the subscription distributor. We interpret these variations as reflecting variations of contractual powers of the players. There are four possible combinations and we study all four as cases 1-4 in the chapter. Table 3.1 corresponds to the 4 different cases depending on the contractual-power of the decision makers.

Case 1: Content provider sets w and K

An all-powerful content provider will drive the pay-per-unit distributor out of business by setting a large wholesale price ($w = \infty$) and earn the centralized profit. The subscription

Table 3.1. Summary of contractual-power: The power of content provider is highest for Case 1 and lowest for Case 4.

		Licensing Contract	
		Content Provider	Subscription Distributor
Wholesale Price Contract	Content Provider	**Case 1**	**Case 2**
	Pay-per-unit Distributor	**Case 3**	**Case 4**

distributor will then charge the monopolist subscription price to optimize the revenues, however all those revenues are extracted by the content provider using the licensing fee K. Therefore, this market achieves coordination, i.e., total profits equal the centralized profit. With a licensing contract coordination is possible as it acts like a "transfer price" in standard supply chain settings. However, with wholesale price we have double marginalization and the system is not coordinated.

Proposition 15. Powerful content provider eliminates pay-per-unit distributor from market *i) When the content provider decides* (w, K)*; then she sets* $w = \infty$ *and* $K = \Pi_{sub}^{mon}$ *to extract all profits from subscription distributor.*
ii) Subscription distributor charges the monopoly price: p_{sub}^{mon}.

Case 2: Content provider sets w and the subscription distributor sets K

In this case, since the subscription distributor is setting the licensing fee K, he will set it to zero, meaning that the content provider will not get any revenues from the subscription retailer. However, she can still affect the equilibrium subscription price by selecting the wholesale price w. If she sets $w = 0$, there will be price wars and nobody will make any profit. If she sets $w = \infty$, she will drive the pay-per-unit distributor out of business and in return she will not make any profits. We identify three scenarios.

Proposition 16. *a) Scenario 1: Low heterogeneity* $(a_1 > 2a_2/3)$*: Subscription distributor gets 100% market penetration, and both the pay-per-unit distributor and the content provider make zero profit;*

b) Scenario 2: Medium heterogeneity ($\sqrt[3]{2a_2} \leq a_1 \leq 2a_2/3$): There may be multiple equilibria, but all players make positive profit;

c) Scenario 3: High heterogeneity ($a_1 \leq \sqrt[3]{2a_2}$): There is a unique equilibrium and all players make positive profit.

Proposition 16 says that if the consumers for experiential products are highly heterogenous $a_1 \leq \sqrt[3]{2}a_2$, then there is a unique equilibrium in market prices and the distributors co-exist without any price wars. In sharp contrast, when the consumers have low heterogeneity $a_1 > \frac{2a_2}{3}$, then subscription distributoxr sells to all consumers in the market and pay-per-unit distributor has zero market share and revenues. In a sense, we have the same result as before in terms of the market penetration. However, subscription distributor gets all the profits from subscription sales, but the profit from pay-per-unit sales are shared between the content provider and the pay-per-unit distributor.

Case 3: Pay-per-unit distributor sets w and content provider sets K

In this case, since the content provider is setting the licensing fee K, she will extract all the revenues from the subscription distributor. On the other hand, the pay-per-unit distributor is setting the wholesale price w and one might expect that she would set $w = 0$, in order not to share any revenues with the content provider. We see that this is not necessarily the case. The intuition is that if the pay-per-unit distributor sets $w = 0$, there are price wars and nobody makes any profit. However there are settings where the pay-per-unit distributor can make positive profits by sharing some of its revenue with the content provider. We again study three scenarios (the ones from Case 2):

Proposition 17. *a) Scenario 1: Low heterogeneity ($a_1 > 2a_2/3$): Subscription distributor gets 100% market penetration, and both the pay-per-unit distributor and the content provider make zero profit;*

b) Scenario 2: Medium heterogeneity ($\sqrt[3]{2a_2} \leq a_1 \leq 2a_2/3$): There may be multiple equilibria, but all players make positive profit;

c) Scenario 3: High heterogeneity ($a_1 \leq \sqrt[3]{2a_2}$): There is a unique equilibrium with a positive wholesale price set by the pay-per-unit distributor. All players make positive profit.

Case 4: Pay-per-unit distributor sets w and subscription distributor sets K

In this case, the analysis is essentially identical to that under case 3. The only difference is that the revenues resulting from subscription sales remain with the subscription distributor, instead of going to the content provider.

Other Contract Types

When the pay-per-unit distributor has a revenue sharing contract f, i.e., a fraction f of the revenues is shared with the content provider, and the subscription based distributor has a licensing contract (K), even then the competition leads to a price war. This is because f and K do not impact the second stage dynamics of the game. Equilibrium revenue sharing contract is $f^e = 0$ if the distributor is *strong* and $f^e = 1$ if the content provider is *strong*. On the other hand, wholesale price contract as studied in the previous section can result in an equilibrium where all (or at least one) players make positive profits. Therefore, a pay-per-unit distributor operating under a wholesale price contract and a subscription distributor operating under a licensing contract creates enough differentiation for experiential products between the distributors that leads to a stable equilibrium in prices.

3.5. Competitive Pricing Strategy for Entrants

So far we discussed competitive pricing between two distributors with different pricing modalities in the market. Now we consider a different situation when there is an *incumbent* distributor using a certain pricing modality (either pay-per-unit or subscription pricing), and a new *entrant* is about to enter the market. If the entrant is free to choose between subscription or pay-per-unit pricing, what should be her optimal strategy? Figure 3.6 shows the sequence of events.

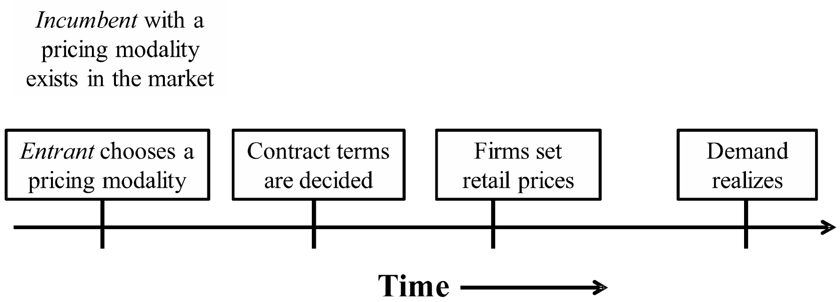


Figure 3.6. Sequence of events when a distributor enters the market to compete with an incumbent.

The impact of the pricing modality of an entrant is summarized in Table 3.2. When the distributors choose different pricing modalities, we end up in one of Cases 1 through 4. However, when both the distributors choose same pricing modality, they end up in price wars.

A powerful content provider allows an entrant to co-exist only with a different pricing modality than the incumbent. If a weak incumbent adopts subscription pricing, then a weak entrant does not enter the market even with a different pricing modality as the content provider sets a high wholesale price driving the entrant out of the market. Interestingly, the decision rule for an entrant is simply defecting from the pricing modality of the incumbent, i.e., choose subscription when the incumbent charges pay-per-unit and vice-versa. When the content provider is all powerful and the incumbent uses subscription pricing, the entrant with a pay-per-unit pricing modality is driven away by the content provider by setting a very high wholesale price (Case 1). Hence, a weak entrant with pay-per-unit is not a credible threat to the incumbent with subscription pricing who is favored by a strong content provider. In all other cases, an entrant is a credible threat to the incumbent, i.e., it takes away consumers from the incumbent. Note that when both distributors are weak, and adopt pay-per-unit pricing, there are multiple subgame perfect equilibria and there is no clear winner between the two pricing modalities.

Table 3.2. An entrant always chooses a complementary pricing scheme to the incumbent. † is discussed in Appendix C.

		Strong Entrant		Weak Entrant	
	Modality	Subscription	Pay-per-unit	Subscription	Pay-per-unit
Strong Incumbent	Subscription	Price war	Case 4	Price war	Case 2
	Pay-per-unit	Case 4	Price war	Case 3	Price war
Weak Incumbent	Modality	Subscription	Pay-per-unit	Subscription	Pay-per-unit
	Subscription	Price war	Case 3	Price war	Case 1
	Pay-per-unit	Case 2	Price war	Case 1	†

Proposition 18. *a) Except for the case of a weak pay-per-unit incumbent and a weak entrant, the pricing modality choice of the entrant is unique and a different modality than the incumbent.*
b) If both distributors are weak then the entrant makes zero profit either way in all equilibria.

In particular, a distributor would enter the market using a different modality than the incumbent. The intuition is that if there are two distributors with the same pricing modality in the market, price wars ensue. Except for the case of a weak pay-per-unit incumbent and a weak entrant, the pricing modality choice of the entrant is unique.

3.6. Conclusions

Experiential products such as movies, music, and books are increasingly sold online with an almost infinite capacity and zero marginal costs. This results in a minimal differentiation among distributors selling these products, who then resort to different pricing modalities to create value for their consumers. Pricing modality is a long-run decision and the distributor's business model revolves around that while in the short-run distributor can adjust its prices to match the fluctuating demand or poach consumers from a competing distributor. The decision to choose the correct pricing modality and the price is not straight forward and gets cumbersome due to competition and contracts with the content provider.

First, we show that a monopolist selling experiential products benefits using subscription pricing modality over pay-per-unit pricing or both subscription and pay-per-unit pricing. We prove this result for a vertically integrated monopolist distributor with no marginal cost of production.

Second, we show that when the wholesale price is positive there is unique stable market equilibrium in prices. When the content provider is all powerful, she drives the pay-per-unit distributor out of the market by imposing a high wholesale price and then the subscription distributor charges the monopoly price to consumers.

Third, we show that in supply chains that distribute experiential products, contractual agreements are set drastically different from conventional supply chains that distribute physical products. For example, in a conventional supply chain, when a distributor sets the wholesale price with a manufacturer, she always sets it to zero (when there is no competition or competing retailers share the same pricing modality). However, due to difference in pricing modalities, i.e., pay-per-unit vs. subscription, as well as a two-stage setting where distributors set the prices, we observe that the pay-per-unit distributor in our setting does not set the wholesale price at zero. She does so to avoid a price war with the subscription distributor. In addition, we show that when the contractual-power is shared among the distributors instead of the content provider, the consumers have a higher surplus and the distributors profitably co-exist.

Finally, we endogenize the pricing modality decision of an entrant. We investigate different scenarios such as a weak incumbent with pay-per-unit pricing, and a strong entrant. We show that it is optimal for an entrant to choose a different pricing modality than the incumbents pricing modality to avoid price wars and maximize her profit.

In the present work, we have studied pay-per-unit and subscription pricing modalities under a stylized setting which restricts the extent to which the findings of our work can be stretched. The consumer behavior framework proposed in the chapter can serve as a basis for interesting future studies: study of different pricing modalities under different contract structures, capacity and service constraint of distributors, different market structures etc.

APPENDIX A

PROOFS FOR CHAPTER 1

Proof of Theorem 1 We discuss only product 1. a) The proof follows from the discussion above the theorem statement.

b) We can check the generalized failure rate at $p = a_1$ and $p \to \infty$.

$$\Lambda^1(p = a_1) = a_1 \frac{dW^1/dp|_{p=a_1}}{1 - W^1(a_1)} = a_1(0/1) = 0 \quad \text{and} \quad \lim_{p\to\infty} \Lambda^1(p) = \infty,$$

where the limit diverges because Λ^1 is increasing. $p/(p - c_1)$ is strictly decreasing in p. This along with $\Lambda^1(p = 0) = 0$ and continuity of Λ^1 implies that Λ^1 crosses $p/(p - c_1)$ exactly at once.

c) Let the unique root of $\Lambda^1(p) = p/(p-c_1)$ be $p_r \in [a_1, b_1]$. Since the profit is increasing at a_1 and has a continuous derivative, it increases over $[a_1, p_r]$. At p_r the derivative is zero. After p_r the derivative must be negative. If the derivative is zero, the root is not unique, which contradicts with the theorem hypothesis. If this derivative is positive, $\Pi_1(p^1 = b_1, p^2) = 0$ implies the existence of another root and contradicts with the theorem hypothesis. This establishes that the derivative is positive until p_r and negative immediately after p_r. If the derivative switches from negative to positive for $p > p_r$, continuity of the derivative implies another root and yields a contradiction. Thus the derivative remains non-positive so the profit decreases for $p > p_r$. Combining this with the fact that the profit increases until p_r, we obtain that the profit is unimodal with maximizer $\Lambda^1(p_r) = p_r/(p_r - c_1)$. $\square$

Proof of Lemma 1 a) We first need the probabilities that show up in the profit expression. For $X \sim \square[0, \epsilon]$ and $\epsilon \leq p^1 \leq b$,

$$\begin{aligned} \mathrm{P}(W^1 \geq p^1) &= \int_0^b \frac{1}{b} \mathrm{P}(X \geq p^1 - x) dx \\ &= \int_{p^1-\epsilon}^{p^1} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} dx + \int_{p^1}^{b} \frac{1}{b} 1 dx = \frac{\epsilon + 2(b - p^1)}{2b}. \end{aligned}$$

Similarly,

$$\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = \int_{p^1-\epsilon}^{p^1} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} \mathrm{P}(X \leq p^2 - x) dx + \int_{p^1}^{b} \frac{1}{b} 1 \mathrm{P}(X \leq p^2 - x) dx.$$

The first and second integrals above depend on the relationship between p^1 and p^2, which can have one of these 4 possible relationships: (2.1) $p^2 \leq p^1 - \epsilon$, (2) $p^1 - \epsilon \leq p^2 \leq p^1$, (3) $p^2 - \epsilon \leq p^1 \leq p^2$, and (4) $p^1 \leq p^2 - \epsilon$. Under case (1), we have $\mathrm{P}(X \leq p^2 - x) = 0$ so $\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = 0$. For case (2), the second integral is zero, and we have

$$\begin{aligned} \mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) &= \int_{p^1-\epsilon}^{p^1} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} \mathbb{I}_{x \leq p^2} \frac{p^2 - x}{\epsilon} dx = \int_{p^1-\epsilon}^{p^2} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} \frac{p^2 - x}{\epsilon} dx \\ &= \frac{(p^2 - p^1 + \epsilon)^3}{6b\epsilon^2}. \end{aligned}$$

For case (3),

$$\begin{aligned} \mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) &= \int_{p^1-\epsilon}^{p^2-\epsilon} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} dx + \int_{p^2-\epsilon}^{p^1} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} \frac{p^2 - x}{\epsilon} dx \\ &\quad + \int_{p^1}^{b} \frac{1}{b} \frac{p^2 - x}{\epsilon} dx \\ &= \frac{(p^1 - p^2)^3 + 3(p^1 - p^2)^2 \epsilon - 3(p^1 - p^2)\epsilon^2 + \epsilon^3}{6b\epsilon^2}. \end{aligned}$$

Finally in case (4),

$$\begin{aligned} \mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) &= \int_{p^1-\epsilon}^{p^1} \frac{1}{b} \frac{\epsilon - (p^1 - x)}{\epsilon} 1 dx + \int_{p^1}^{p^2-\epsilon} \frac{1}{b} 1 dx + \int_{p^2-\epsilon}^{p^2} \frac{1}{b} \frac{p^2 - x}{\epsilon} dx \\ &= \frac{p^2 - p^1}{b}. \end{aligned}$$

b) Retailer 1's duopoly profit maximization problem for retailer 2's price p^2 is

$$\max_p (p-c_1)\{\phi\frac{(\epsilon+2(b-p))}{2b}+(1-\phi)\big[\mathbb{1}_{\{p^2\le p\le p^2+\epsilon\}}\frac{(p^2-p+\epsilon)^3}{6b\epsilon^2}$$
$$+\mathbb{1}_{\{p^2-\epsilon\le p\le p^2\}}\frac{(p-p^2)^3+3(p-p^2)^2\epsilon-3(p-p^2)\epsilon^2+\epsilon^3}{6b\epsilon^2}+\mathbb{1}_{\{p^1\le p^2-\epsilon\}}\frac{p^2-p}{b}\big]\}.$$

We can find the best response price p^1 for retailer 1 as

$$p^1=\frac{2c_1+2b+\epsilon}{4} \text{ if } p^2\le p^1-\epsilon,$$

$$3(p^1-c_1)\left((-p^1+p^2+\epsilon)^2(-1+\phi)-2\phi\epsilon^2\right) = (p^1-p^2-\epsilon)^3(1-\phi) \quad +3(2(p^1-b)-\epsilon)\phi\epsilon^2$$
$$\text{if } p^1-\epsilon\le p^2\le p^1, \tag{A.1}$$

$$\left((p^1-p^2)^3+3(p^1-p^2)^2\epsilon-3(p^1-p^2)\epsilon^2+\epsilon^3\right)(1-\phi)+3\epsilon^2(2b-2p^1+\epsilon)\phi$$
$$+(p^1-c_1)\left(3(p^1-p^2)(p^1-p^2+2\epsilon)(1-\phi)-3\epsilon^2(1+\phi)\right)=0 \text{ if } p^2-\epsilon\le p^1\le p^2, \tag{A.2}$$

$$p^1=\frac{\phi(\epsilon+2b)+2(1-\phi)p^2+2c_1}{4} \text{ if } p^1\le p^2-\epsilon.$$

Retailer 2's profit maximization problem and the best response can be obtained similarly. (A.1) and (A.2) are cubic in p^1 and obtaining the equilibrium for non-identical retailers requires some algebra.

Assuming identical retailers, i.e., $c_1=c_2=c$ and $\phi=1-\phi=1/2$. We can identify all possible equilibria. At the symmetric equilibrium, i.e., $p^{1e}=p^{2e}$, (A.1) and (A.2) simplify to the same linear equation in p^1 and we obtain $p^{1e}=p^{2e}=\frac{6b+9c+4\epsilon}{15}$. Therefore, the symmetric equilibrium is $(p^{1e},p^{2e})=\left(\frac{6b+9c+4\epsilon}{15},\frac{6b+9c+4\epsilon}{15}\right)$. We find 2 non-symmetric equilibria: $(p^{1e},p^{2e})=\left(\frac{8b+8c+4\epsilon}{16},\frac{6b+10c+3\epsilon}{16}\right)$ when $p^{2e}\le p^{1e}-\epsilon$ and $(p^{1e},p^{2e})=\left(\frac{6b+10c+3\epsilon}{16},\frac{8b+8c+4\epsilon}{16}\right)$ when $p^{1e}\le p^{2e}-\epsilon$. □

Proof of Theorem 2 a) Let

$$\begin{aligned}
r_a &= \mathrm{P}(p_l^1 \le W^1 < p_h^1, p_l^2 \le W^2 < p_h^2); & r_b &= \mathrm{P}(p_l^1 \le W^1 < p_h^1, p_h^2 \le W^2);\\
r_c &= \mathrm{P}(p_h^1 \le W^1, p_l^2 \le W^2 < p_h^2); & r_d &= \mathrm{P}(p_h^1 \le W^1, p_h^2 \le W^2);\\
r_e &= \mathrm{P}(p_l^1 \le W^1 < p_h^1, p_l^2 > W^2); & r_f &= \mathrm{P}(p_h^1 \le W^1, p_l^2 > W^2);\\
r_g &= \mathrm{P}(p_l^1 > W^1, p_l^2 \le W^2 < p_h^2); & r_h &= \mathrm{P}(p_l^1 > W^1, p_h^2 \le W^2),
\end{aligned}$$

then, by (1.8), the profits are

$$\begin{aligned}
\Pi^1(p_l^1, p_l^2) &= (p_l^1 - c_1)[\phi(r_a + r_b + r_c + r_d + r_e + r_f) + (1-\phi)(r_e + r_f)];\\
\Pi^1(p_l^1, p_h^2) &= (p_l^1 - c_1)[\phi(r_a + r_b + r_c + r_d + r_e + r_f) + (1-\phi)(r_a + r_c + r_e + r_f)];\\
\Pi^1(p_h^1, p_l^2) &= (p_h^1 - c_1)[\phi(r_c + r_d + r_f) + (1-\phi)r_f];\\
\Pi^1(p_h^1, p_h^2) &= (p_h^1 - c_1)[\phi(r_c + r_d + r_f) + (1-\phi)(r_c + r_f)];\\
\Pi^2(p_l^1, p_l^2) &= (p_l^2 - c_2)[(1-\phi)(r_a + r_b + r_c + r_d + r_g + r_h) + \phi(r_g + r_h)];\\
\Pi^2(p_h^1, p_l^2) &= (p_l^2 - c_2)[(1-\phi)(r_a + r_b + r_c + r_d + r_g + r_h) + \phi(r_a + r_b + r_g + r_h)];\\
\Pi^2(p_l^1, p_h^2) &= (p_h^2 - c_2)[(1-\phi)(r_b + r_d + r_h) + \phi r_h];\\
\Pi^2(p_h^1, p_h^2) &= (p_h^2 - c_2)[(1-\phi)(r_b + r_d + r_h) + \phi(r_b + r_h)].
\end{aligned}$$

A price cycle $\{p_l^1, p_l^2\} \to \{p_l^1, p_h^2\} \to \{p_h^1, p_h^2\} \to \{p_h^1, p_l^2\} \to \{p_l^1, p_l^2\}$ of length 4 implies 2 inequalities on retailer 1 profits $\Pi^1(p_l^1, p_l^2) > \Pi^1(p_h^1, p_l^2)$, $\Pi^1(p_l^1, p_h^2) < \Pi^1(p_h^1, p_h^2)$ and 2 inequalities on retailer 2 profits $\Pi^2(p_l^1, p_l^2) < \Pi^2(p_l^1, p_h^2)$, $\Pi^2(p_h^1, p_l^2) > \Pi^2(p_h^1, p_h^2)$.

The inequalities on retailer 1 profits imply $(p_l^1 - c_1)(\phi(r_a + r_b + r_c + r_d) + r_e + r_f) > (p_h^1 - c_1)(r_f + \phi(r_c + r_d))$ and $(p_l^1 - c_1)(\phi(r_b + r_d) + r_a + r_c + r_e + r_f) < (p_h^1 - c_1)(r_f + r_c + \phi r_d)$. These two inequalities lead to

$$\begin{aligned}
1 + \frac{r_e + \phi(r_a + r_b)}{r_e + r_f + \phi(r_a + r_b + r_c + r_d)} - \frac{r_a + r_e + \phi r_b}{r_a + r_c + r_e + r_f + \phi(r_b + r_d)}\\
> \frac{p_l^1 - c_1}{p_h^1 - c_1} + \frac{r_e + \phi(r_a + r_b)}{r_e + r_f + \phi(r_a + r_b + r_c + r_d)} > 1, \quad \text{(A.3)}
\end{aligned}$$

and using the terms that do not depend on prices

$$\frac{r_e+\phi(r_a+r_b)}{r_e+r_f+\phi(r_a+r_b+r_c+r_d)} > \frac{r_a+r_e+\phi r_b}{r_a+r_c+r_e+r_f+\phi(r_b+r_d)}.$$

This inequality implies $r_a(r_f+\phi r_d) > r_c(r_e+\phi r_b)$. Similarly, the inequalities on retailer 2 profits imply $(p_l^2-c_2)((1-\phi)(r_a+r_b+r_c+r_d)+r_g+r_h) < (p_h^2-c_2)(r_h+(1-\phi)(r_b+r_d))$ and $(p_l^2-c_2)((1-\phi)(r_c+r_d)+r_a+r_b+r_g+r_h) > (p_h^2-c_2)(r_h+r_b+(1-\phi)r_d)$. Then

$$\begin{aligned}1-\frac{r_a+r_g+(1-\phi)r_c}{r_a+r_b+r_g+r_h+(1-\phi)(r_c+r_d)}+\frac{r_g+(1-\phi)(r_a+r_c)}{r_g+r_h+(1-\phi)(r_a+r_b+r_c+r_d)}&\\ <\frac{p_l^2-c_2}{p_h^2-c_2}+\frac{r_g+(1-\phi)(r_a+r_c)}{r_g+r_h+(1-\phi)(r_a+r_b+r_c+r_d)}<1&,\\ 1+\frac{r_a+r_g+(1-\phi)r_c}{r_a+r_b+r_g+r_h+(1-\phi)(r_c+r_d)}-\frac{r_g+(1-\phi)(r_a+r_c)}{r_g+r_h+(1-\phi)(r_a+r_b+r_c+r_d)}&\\ >\frac{p_l^2-c_2}{p_h^2-c_2}+\frac{r_a+r_g+(1-\phi)r_c}{r_a+r_b+r_g+r_h+(1-\phi)(r_c+r_d)}>1&\end{aligned} \quad \text{(A.4)}$$

and using the terms that do not depend on prices

$$\frac{r_a+r_g+(1-\phi)r_c}{r_a+r_b+r_g+r_h+(1-\phi)(r_c+r_d)} > \frac{r_g+(1-\phi)(r_a+r_c)}{r_g+r_h+(1-\phi)(r_a+r_b+r_c+r_d)}.$$

This inequality implies $r_b(r_g+(1-\phi)r_c) > r_a(r_h+(1-\phi)r_d)$.

For the cycle, it is necessary to have

$$r_c(r_e+\phi r_b) < r_a(r_f+\phi r_d) \;\text{ and }\; r_b(r_g+(1-\phi)r_c) > r_a(r_h+(1-\phi)r_d).$$

When $r_ar_f=r_cr_e$ and $r_ar_h=r_br_g$, one of these inequalities fail and $\{p_l^1,p_l^2\}\to\{p_l^1,p_h^2\}\to\{p_h^1,p_h^2\}\to\{p_h^1,p_l^2\}\to\{p_l^1,p_l^2\}$ cannot be a price cycle. The condition $r_ar_f=r_cr_e$ can be written as $r_f(r_a+r_c+r_e+r_f)=(r_e+r_f)(r_c+r_f)$, i.e., $\mathrm{P}(p_h^1\le W^1,W^2<p_l^2)\mathrm{P}(p_l^1\le W^1,W^2<p_h^2)=\mathrm{P}(p_l^1\le W^1,W^2<p_l^2)\mathrm{P}(p_h^1\le W^1,W^2<p_h^2)$. Combining this with the analogous equality obtained from $r_ar_h=r_br_g$, we obtain

$$\begin{aligned}&\mathrm{P}(W^i\ge p_h^i,\ W^{-i}<p_l^{-i})\mathrm{P}(W^i\ge p_l^i,\ W^{-i}<p_h^{-i})\\ &\qquad=\mathrm{P}(W^i\ge p_l^i,\ W^{-i}<p_l^{-i})\mathrm{P}(W^i\ge p_h^i,\ W^{-i}<p_h^{-i}).\end{aligned} \quad \text{(A.5)}$$

When we consider $\{p_l^1, p_l^2\} \leftarrow \{p_l^1, p_h^2\} \leftarrow \{p_h^1, p_h^2\} \leftarrow \{p_h^1, p_l^2\} \leftarrow \{p_l^1, p_l^2\}$ as a price cycle (reverse in direction to the price cycle studied above), the inequalities in (A.3) for retailer 1 reverses the direction. The proof is similar to above and we again obtain the same equality condition on probabilities $r_a r_f = r_c r_e$ and $r_a r_h = r_b r_g$. Therefore (A.5) eliminates also the cycle $\{p_l^1, p_l^2\} \leftarrow \{p_l^1, p_h^2\} \leftarrow \{p_h^1, p_h^2\} \leftarrow \{p_h^1, p_l^2\} \leftarrow \{p_l^1, p_l^2\}$.

b) Setting $p_l^i = a_i$ and $p_h^{-i} = b_{-i} + \epsilon$ for small ϵ, (A.5) becomes $\mathrm{P}(W^i \geq p_h^i,\ W^{-i} < p_l^{-i}) = \mathrm{P}(W^{-i} < p_l^{-i})\mathrm{P}(W^i \geq p_h^i)$. When this condition is satisfied for all prices, W^i and W^{-i} are independent.

c) When the cycle includes (p^1, p^2) and p^i is equal to the lowest WTP for retailer i, probabilities r_e, r_f, r_g, r_h all become zero. Then $r_a r_f = r_c r_e$ and $r_a r_h = r_b r_g$, and profit inequalities guaranteeing the cycle fail, so no cycle of length 4 exists. When the prices are binary, the price cycle must include the lowest WTPs for both retailers. This is impossible and no cycle can exist, so there must be a price-pair equilibrium.

d) For retailer i, $W^i \in [a_i, b_i)$ and hence $a_i \leq \{p_l^i, p_h^i\} \leq b_i$. First we consider profit inequalities of retailer 1 and from (A.3) we have

$$1 - \frac{r_a + r_e + \phi r_b}{r_a + r_c + r_e + r_f + \phi(r_b + r_d)} > \frac{p_l^1 - c_1}{p_h^1 - c_1} \geq \frac{a_1 - c_1}{b_1 - c_1}.$$

Therefore $\{p_l^1, p_l^2\} \to \{p_l^1, p_h^2\} \to \{p_h^1, p_h^2\} \to \{p_h^1, p_l^2\} \to \{p_l^1, p_l^2\}$ cannot be a price cycle if

$$\begin{aligned} \frac{r_c + r_f + \phi r_d}{r_a + r_e + \phi r_b} &\leq \frac{a_1 - c_1}{b_1 - a_1}, \\ \frac{\mathrm{P}(p_h^1 \leq W^1, W^2 < p_h^2) + \phi \mathrm{P}(p_h^1 \leq W^1, p_h^2 \leq W^2)}{\mathrm{P}(p_l^1 \leq W^1 < p_h^1, W^2 < p_h^2) + \phi \mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_h^2 \leq W^2)} &\leq \frac{a_1 - c_1}{b_1 - a_1}. \end{aligned}$$

There is no price cycle of length 4 when the above condition is satisfied for all the prices $\{p_l^1, p_h^1, p_h^2\}$

$$\max_{\{p_l^1, p_h^1, p_h^2\}} \left\{ \frac{\mathrm{P}(p_h^1 \leq W^1, W^2 < p_h^2) + \phi \mathrm{P}(p_h^1 \leq W^1, p_h^2 \leq W^2)}{\mathrm{P}(p_l^1 \leq W^1 < p_h^1, W^2 < p_h^2) + \phi \mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_h^2 \leq W^2)} \right\} \leq \frac{a_1 - c_1}{b_1 - a_1}.$$

When we consider the opposite price cycle,i.e., $\{p_l^1, p_l^2\} \leftarrow \{p_l^1, p_h^2\} \leftarrow \{p_h^1, p_h^2\} \leftarrow \{p_h^1, p_l^2\} \leftarrow \{p_l^1, p_l^2\}$, we have from (A.3) similar to above

$$1 - \frac{r_e + \phi(r_a + r_b)}{r_e + r_f + \phi(r_a + r_b + r_c + r_d)} > \frac{p_l^1 - c_1}{p_h^1 - c_1} \geq \frac{a_1 - c_1}{b_1 - c_1}.$$

Therefore $\{p_l^1, p_l^2\} \leftarrow \{p_l^1, p_h^2\} \leftarrow \{p_h^1, p_h^2\} \leftarrow \{p_h^1, p_l^2\} \leftarrow \{p_l^1, p_l^2\}$ cannot be a price cycle if

$$\begin{aligned} \frac{r_f + \phi(r_c + r_d)}{r_e + \phi(r_a + r_b)} &\leq \frac{a_1 - c_1}{b_1 - a_1}, \\ \frac{\mathrm{P}(p_h^1 \leq W^1, p_l^2 > W^2) + \phi \mathrm{P}(p_h^1 \leq W^1, p_l^2 \leq W^2)}{\mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_l^2 > W^2) + \phi \mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_l^2 \leq W^2)} &\leq \frac{a_1 - c_1}{b_1 - a_1}. \end{aligned}$$

There is no price cycle of length 4 when the above condition is satisfied for all the prices $\{p_l^1, p_h^1, p_l^2\}$

$$\max_{\{p_l^1, p_h^1, p_l^2\}} \left\{ \frac{\mathrm{P}(p_h^1 \leq W^1, p_l^2 > W^2) + \phi \mathrm{P}(p_h^1 \leq W^1, p_l^2 \leq W^2)}{\mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_l^2 > W^2) + \phi \mathrm{P}(p_l^1 \leq W^1 < p_h^1, p_l^2 \leq W^2)} \right\} \leq \frac{a_1 - c_1}{b_1 - a_1}.$$

Subsequently the condition to rule the cycles out with retailer 1's profit is generalized as

$$\max_{\{p_l^1, p_h^1, p^2\}} \left\{ \frac{\mathrm{P}(p_h^1 \leq W^1, p^2 > W^2) + \phi \mathrm{P}(p_h^1 \leq W^1, p^2 \leq W^2)}{\mathrm{P}(p_l^1 \leq W^1 < p_h^1, p^2 > W^2) + \phi \mathrm{P}(p_l^1 \leq W^1 < p_h^1, p^2 \leq W^2)} \right\} \leq \frac{a_1 - c_1}{b_1 - a_1}.$$

Similarly, we can obtain a condition for retailer 2's profits using (A.4). There is no price cycle of length 4 if, for either $i = 1$ or $i = 2$, the following condition is satisfied for either $i = 1$ or 2

$$\max_{\{p_l^i, p_h^i, p^{-i}\}} \left\{ \frac{\mathrm{P}(p_h^i \leq W^i, p^{-i} > W^{-i}) + \phi_i \mathrm{P}(p_h^i \leq W^i, p^{-i} \leq W^{-i})}{\mathrm{P}(p_l^i \leq W^i < p_h^i, p^{-i} > W^{-i}) + \phi_i \mathrm{P}(p_l^i \leq W^i < p_h^i, p^{-i} \leq W^{-i})} \right\} \leq \frac{a_i - c_i}{b_i - a_i}.$$

e) When either retailer charges binary prices, the only possible price cycles are of length 4. However, due to Theorem 2.c there is no price cycle of length 4 and hence there are no price cycles. Therefore, there must be a price-pair equilibrium. □

Proof of Theorem 3 We know that the retailer's price response when preference $\phi(p^1, p^2)$ is dependent on prices (p^1, p^2) is given from (1.9). It is easy to see that when $\partial\phi(p^1, p^2)/\partial p^1 = 0$ and $\partial\phi(p^1, p^2)/\partial p^2 = 0$, we obtain price responses when the preference is independent of the prices given by (1.6). When preference ϕ is increasing in p^1 the best response satisfies

$$\begin{aligned} p^1 &= c_1 + \left[\frac{\Lambda^1(p^1)}{p^1} - \frac{(1 - W^2(p^2))[\partial\phi(p^1, p^2)/\partial p^1]}{\{\phi(p^1, p^2) + (1 - \phi(p^1, p^2))W^2(p^2)\}} \right]^{-1} \\ &> c_1 + \left[\Lambda^1(p^1)/p^1 \right]^{-1}. \end{aligned} \tag{A.6}$$

The right-hand side is the price response of retailer 1 when ϕ is independent of prices. Similarly, we can prove that retailer 1 responds with a lower price if the preference ϕ is decreasing in p^1, compared to the price offered when ϕ is independent of prices.

Let the equilibrium without price dependent preferences be $(p^{1,e}, p^{2,e})$. When price responses $p^1(p^2)$ and $p^2(p^1)$ both increase due to price dependent preferences, we have $p^1(p^2) \geq p^{1,e}$ for every p^2 and $p^2(p^1) \geq p^{2,e}$ for every p^1. Then these responses can only intersect at a point (p^1, p^2) such that $p^1 \geq p^{1,e}$ and $p^2 \geq p^{2,e}$, so the equilibrium with price dependence has higher prices than the equilibrium without price dependence. □

Proof of Lemma 2 a) Price p^i_{bo} solves $x - c_i - \nu_i d_i = \frac{1-W^i(x)}{dW^i(x)/dx}$ while p^i_{ls} solves $x - c_i = \frac{1-W^i(x)}{dW^i(x)/dx}$. By the virtue of IFR property of W^i, we have the right-hand side in both equations decreasing in x and the left-hand side linear in x. Using $\nu_i d_i \geq 0$, we see from Figure A.1, points $(p^i_{ls}, p^i_{ls} - c_i)$, $(p^i_{ls}, p^i_{ls} - c_i - \nu_i d_i)$ and $(p^i_{ls} + \nu_i d_i, p^i_{ls} - c_i)$ define a 90-degree triangle. Since $(1 - W^i)/dW^i$ enters the triangle at $(p^i_{ls}, p^i_{ls} - c_i)$ and continues to decrease, it must exit the triangle from the hypotenuse. The x-axis of the exit point gives p^i_{bo}, which satisfies the bounds in the lemma.

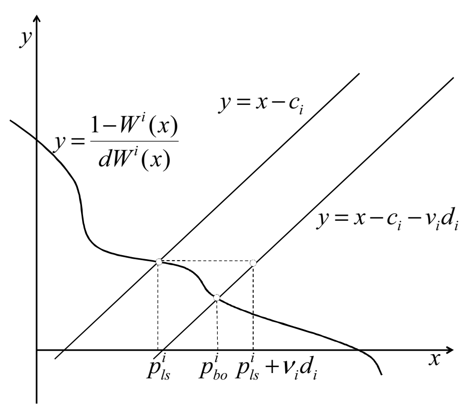


Figure A.1. Lost sales and backorder price response curves.

b) The backorder price maximizes an objective proportional to $(p^i - c_i - \nu_i d_i)(1 - W^i(p^i))$ when the customers do not receive $\nu_i d_i$. If customers receive $\nu_i d_i$, the objective is proportional to $(p^i - c_i - \nu_i d_i)(1 - W^i(p^i - \nu_i d_i))$, or proportional to $(\bar{p}^i - c_i)(1 - W^i(\bar{p}^i))$ where $\bar{p}^i = p^i - \nu_i d_i$. The lost sales objective is proportional to $(p^i - c_i)(1 - W^i(p))$ and has the same functional form as $(\bar{p}^i - c_i)(1 - W^i(\bar{p}^i))$, so it has the same maximizer as $(\bar{p}^i - c_i)(1 - W^i(\bar{p}^i))$: p^i_{ls} is the effective price $p^i - \nu_i d_i$ charged when customers backorder and are compensated by $\nu_i d_i$. □

Proof of Lemma 3 Since setting the same prices p^1_{bo} and p^2_{bo} is an equilibrium for both profits Π^i_{ava} and Π^i_{ret}, we can suppress the prices and write profits as $\Pi^{i,e}_{ret}(\nu_i)$ and $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i})$. We have

$$\begin{aligned}
\Pi^{i,e}_{ava}(\nu_i, \nu_{-i} = 0) &= (p^i_{bo} - c_i - \nu_i d_i)\left[1 - W^i\right](\phi_{-i} W^{-i} + (1 - \nu_i)\phi_i + \nu_i \phi_i W^{-i}) \leq \Pi^{i,e}_{ret}(\nu_i), \\
\Pi^{i,e}_{ava}(\nu_i, \nu_{-i} = 1) &= (p^i_{bo} - c_i - \nu_i d_i)\left[1 - W^i\right]((1 - \nu_i)[\phi_i + \phi_{-i}] + \nu_i[\phi_i + \phi_{-i} W^{-i}]) \geq \Pi^{i,e}_{ret}(\nu_i)
\end{aligned}$$

and $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i})$ increases in ν_{-i}. Hence, we can always find $\bar{\nu}_{-i}$ such that $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i}) \geq \Pi^{i,e}_{ret}(\nu_i)$ for $\nu_{-i} \geq \bar{\nu}_{-i}$.

On the other hand, $\Pi^i_{ava}(\nu_i, \nu_{-i})$ and $\Pi^i_{ret}(\nu_i)$ are decreasing in ν_i, and

$$\begin{aligned}
\Pi^{i,e}_{ava}(\nu_i = 0, \nu_{-i}) &= (p^i_{bo} - c_i)\left[1 - W^i\right](\phi_i + \phi_{-i}\left\{(1 - \nu_{-i})W^{-i} + \nu_{-i}\right\}) \geq \Pi^{i,e}_{ret}(\nu_i), \\
\Pi^{i,e}_{ava}(\nu_i = 1, \nu_{-i}) &= (p^i_{bo} - c_i - d_i)\left[1 - W^i\right]\left(\phi_i\left\{\nu_{-i} + (1 - \nu_{-i})W^{-i}\right\} + \phi_{-i} W^{-i}\right) \leq \Pi^{i,e}_{ret}(\nu_i).
\end{aligned}$$

These show that there exist $\bar{\nu}_i$ where $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i}) = \Pi^{i,e}_{ret}(\nu_i)$. To establish the uniqueness of $\bar{\nu}_i$, we use the second order derivative condition for convexity of (1.20) and obtain

$$\frac{\partial^2 \Pi^{i,e}_{ava}(\nu_i, \nu_{-i})}{\partial \nu_i^2} = d_i[1 - W^i][1 - W^{-i}]\left[\phi_i(1 - \nu_{-i}) + \phi_{-i}\nu_{-i}\right] \geq 0.$$

Since $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i})$ is convex and decreasing in ν_i, and $\Pi^i_{ret}(\nu_i)$ is linearly decreasing in ν_i these profits are equal only once at $\bar{\nu}_i$, so $\Pi^{i,e}_{ava}(\nu_i, \nu_{-i}) \geq \Pi^{i,e}_{ret}(\nu_i)$ for $\nu_i \leq \bar{\nu}_i$. Furthermore, we have $\Pi^{i,e}_{ava}(\nu_i = 0, \nu_{-i}) \geq \Pi^{i,e}_{ret}(\nu_i)$ and $\Pi^{i,e}_{ava}(\bar{\nu}_i, \nu_{-i}) = \Pi^{i,e}_{ret}(\bar{\nu}_i)$, therefore $\frac{\partial \Pi^{i,e}_{ava}(\nu_i, \nu_i)}{\partial \nu_i} \leq \frac{\partial \Pi^{i,e}_{ret}(\nu_i)}{\partial \nu_i} \leq 0$ for $\nu_i \leq \bar{\nu}_i$. □

Proof of Lemma 4 From Table 1.4 and no backordering probability in the lost sales case, we have $\rho^i_{ls}(\nu_i) \leq \rho^i_{ava}(\nu_i)$ for every value of ν_i. Also $\rho^i_{ls}(\nu_i)$ decreases in ν_i, specifically,

$0 = \rho^i_{ls}(\nu_i = 1) \leq \rho^i_{ava}(\nu_i = 1) \leq \rho^i_{ret} \leq \rho^i_{ava}(\nu_i = 0) = \rho^i_{ls}(\nu_i = 0)$. Then we can always find $\bar{\nu}^i_{ls} \leq \bar{\nu}^i_{ava}$ such that $\rho^i_{ls}(\nu_i) \geq \rho^i_{ret}$ for $\nu_i \leq \bar{\nu}^i_{ls}$. Combining this with Lemma 3, we obtain Lemma 4. □

Proof of Theorem 4 The profits for lost sales case, retailer favoring and availability favoring cases under $d_i = p^i - c_i$, and monopoly case become as follows.

$$\begin{aligned}
\Pi^i_{ls}(p^1, p^2) &= (1-\nu_i)(p^i - c_i)(1 - W_i(p^i))[\phi_i + \phi_{-i}((1-\nu_{-i})W_{-i}(p^{-i}) + \nu_{-i})] \\
\Pi^i_{ret,-d}(p^1, p^2) &= (1-\nu_i)(p^i - c_i)(1 - W^i(p^i))[\phi_i + \phi_{-i}W^{-i}(p^{-i})] \\
\Pi^i_{ava,-d}(p^1, p^2) &= (1-\nu_i)(p^i - c_i)(1 - W^i(p^i))\Big[(1-\nu_i)\left[\phi_i + \phi_{-i}\left\{(1-\nu_{-i})W^{-i}(p^{-i}) + \nu_{-i}\right\}\right] \\
&\qquad + \nu_i\left[\phi_i\left\{\nu_{-i} + (1-\nu_{-i})W^{-i}(p^{-i})\right\} + \phi_{-i}W^{-i}(p^{-i})\right]\Big] \\
\Pi^i_M(p^i) &= (1-\nu_i)(p^i - c_i)(1 - W_i(p^i))
\end{aligned}$$

In view of $\Pi_{ret,-d}$ and $\Pi_{ava,-d}$, the backorder price in (1.19) changes and is denoted by $p^i_{bo,-d}$. This price solves (1.13) and so $p^i_{bo,-d} = p^i_{ls}$. That is, setting $d_i = p^i - c_i$ equates the backorder equilibrium price to lost sales equilibrium price.

From the profit expressions under lost sales and retailer favoring customers, we immediately have $\Pi^i_{ls}(p^1, p^2) \geq \Pi^i_{ret,-d}(p^1, p^2)$. We can also establish a similar equality between $\Pi^i_{ls}(p^1, p^2)$ and $\Pi^i_{ava,-d}(p^1, p^2)$ as follows. Let us fix $(p^1, p^2; \nu_i)$ and consider both of these profits parametrically as ν_{-i} varies from 0 to 1. We have the following inequalities at $\nu_{-i} = 0$ and $\nu_{-i} = 1$.

$$\begin{aligned}
\Pi^i_{ls}(p^1, p^2; \nu_i, \nu_{-i} = 0) &= (p^i - c_i)(1-\nu_i)(1 - W_i)[\phi_{-i}W_{-i} + \phi_i] \\
&\geq (p^i - c_i)(1-\nu_i)(1 - W_i)[\phi_{-i}W_{-i} + (1-\nu_i)\phi_i + \nu_i W_{-i}\phi_i] \\
&= \Pi^i_{ava,-d}(p^1, p^2; \nu_i, \nu_{-i} = 0).
\end{aligned}$$

$$\begin{aligned}
\Pi^i_{ls}(p^1, p^2; \nu_i, \nu_{-i} = 1) &= (p^i - c_i)(1-\nu_i)(1 - W_i)[\phi_i + \phi_{-i}] \\
&\geq (p^i - c_i)(1-\nu_i)(1 - W_i)[(1-\nu_i)[\phi_i + \phi_{-i}] + \nu_i[\nu_{-i}\phi_i + W_{-i}\phi_{-i}]] \\
&= \Pi^i_{ava,-d}(p^1, p^2; \nu_i, \nu_{-i} = 1).
\end{aligned}$$

Both $\Pi^i_{ls}(p^1, p^2; \nu_i, \nu_{-i})$ and $\Pi^i_{ava,-d}(p^1, p^2; \nu_i, \nu_{-i})$ are linear in ν_{-i}. Combining this with

$$\Pi^i_{ls}(p^1, p^2; \nu_i, \nu_{-i} = 0) \geq \Pi^i_{ava,-d}(p^1, p^2; \nu_i, \nu_{-i} = 0),$$

and

$$\Pi^i_{ls}(p^1, p^2; \nu_i, \nu_{-i} = 1) \geq \Pi^i_{ava,-d}(p^1, p^2; \nu_i, \nu_{-i} = 1),$$

we obtain $\Pi^i_{ls}(p^1, p^2) \geq \Pi^i_{ava,-d}(p^1, p^2)$.

Since the equilibrium prices are all the same, we obtain $\Pi^{i,*}_M \geq \Pi^{i,e}_{ls}$, $\Pi^{i,e}_{ls} \geq \Pi^{i,e}_{ava,-d}$ and $\Pi^{i,e}_{ls} \geq \Pi^{i,e}_{ret,-d}$. These inequalities and the specialization of the profit inequality in Lemma 3 complete the proof. □

Lemma 5. (Concavity of Log-likelihood). *a)* $L_{sl}(\alpha, \beta)$ *is concave in parameters* (α, β).
b) $L_{wtp}(\delta, \Phi, W)$ *is concave in parameters* (δ, Φ).
c) If WTPs are shifted exponential, i.e., $W^m(p) = 1 - \exp(-\tau_m(p - a_m))$, *then* $L_{wtp}(\delta, \Phi, \tau_1, \ldots, \tau_M, a_1, \ldots, a_M)$ *is concave in parameters* τ_m *and* a_m.

Proof a) Concavity of L_{sl} has been established in the literature; for example see pp.105-142 of McFadden (1974). For completeness, we provide a proof, whose steps are also used in b) and c). Let $\theta = [\beta, \alpha_1, \alpha_2, \ldots, \alpha_M]$, $x_0 = [0; \ldots; 0]$, $x_m = [p_m; 0; \ldots; 0; 1; 0; \ldots; 0]$ where 1 is in the $(m+1)$st spot, so θ is a row vector, others are column vectors and the scalar product θx_m is defined. Then we can write the likelihood for a single customer as $L_{sl}(\theta) = \sum_{m=0}^{M} y^m(\theta x_m) - \log \sum_{m=0}^{M} \exp(\theta x_m)$. Ignoring the linear part, we need to show $\log \sum_m \exp(\theta x_m)$ is convex in θ. This follows from three facts. First, $\log \sum_m \exp(z_m)$ is convex in z_m; see p.74 of Boyd and Vandenberghe (2009). Second, $z_m = \theta x_m$ is linear in θ. Third, the composite function $f(g(\theta))$ is convex when f is convex and g is linear.

b) First, we claim $\rho^0 = 1 - \sum_{m=1}^{M} \rho^m = 1 - \delta + \delta \prod_{m=1}^{M} W^m(p^m)$. It is sufficient to prove that a customer interested in one of the products does not buy any with probability $\prod_{m=1}^{M} W^m(p^m)$. Each product m belongs to a consideration set $\mathfrak{L}_i$, and if $w^m \geq p^m$, then either m or another product in $\mathfrak{L}_i$ is purchased. That is, $w^m \geq p^m$ for a product m implies the purchase of a product. The contrapositive of this statement is that no purchase implies $w^m \geq p^m$ for each product m. On the other hand, $w^m \geq p^m$ for each product m also implies no purchase. Hence, $\prod_{m=1}^M W^m(p^m)$ is the probability that an interested customer does not buy a product.

For given W,

$$\begin{aligned} L_{wtp}(\delta, \Phi, W) &= y^0 \left[\log \left(\delta \prod_{m=1}^{M} W^m(p^m) + 1 - \delta \right) \right] \\ &+ \sum_{m=1}^{M} y^m \left[\log \delta + \log(1 - W^m(p^m)) + \log \left(\sum_{i=1}^{S} \phi_i \mathbb{I}_{m \in \mathfrak{L}_i} \prod_{j \in \mathfrak{L}_i^{\leq m}} W^j(p^j) \right) \right]. \end{aligned}$$

Dropping constants and setting $C = \prod_{m=1}^M W^m(p^m)$ and $C_{m,i} = \mathbb{I}_{m \in \mathfrak{L}_i} \prod_{j \in \mathfrak{L}_i^{\leq m}} W^j(p^j)$, we have

$$y^0 \left[\log((C-1)\delta + 1) \right] + \sum_{m=1}^{M} y^m \left[\log \delta + \log \left(\sum_{i=1}^{S} C_{m,i} \phi_i \right) \right].$$

Since the logarithm of concave functions are concave, it suffices to establish concavity of $(C-1)\delta + 1$, δ and $C_{m,i}\phi_i$. These are all linear in parameters (δ, Φ) so $L_{wtp}(\delta, \Phi, W)$ is concave in (δ, Φ).

c) For given (δ, Φ),

$$\begin{aligned} L_{wtp}(\delta, \Phi, W) &= y^0 \left[\log \left(\delta \prod_{m=1}^{M} (1 - \exp(-\tau_m(p^m - a_m))) + 1 - \delta \right) \right] \\ &+ \sum_{m=1}^{M} y^m \Bigg[\log \delta - \tau_m(p^m - a_m) \\ &+ \log \left(\sum_{i=1}^{S} \phi_i \mathbb{I}_{m \in \mathfrak{L}_i} \prod_{j \in \mathfrak{L}_i^{\leq m}} (1 - \exp(-\tau_j(p^j - a_j))) \right) \Bigg]. \end{aligned}$$

Since the logarithm of concave functions are concave, it suffices to establish the concavity of $\Pi_{j \in \mathfrak{L}_i^{\leq m}}(1 - \exp(-\tau_j(p^j - a_j)))$. This follows from the concavity of $1 - \exp(-\tau_j(p^j - a_j))$ in

τ_j and a_j, which follows from three facts. First, $\tau_j(p^j - a_j))$ is linear in parameters. Second, $-\exp(-x)$ is concave. Third, the composite function $f(g(\theta))$ is concave when f is concave and g is linear. We prove concavity in (δ, Φ) in b) and in τ_m and a_m in c), but these do not imply joint concavity in $(\delta, \Phi, \tau_1, \ldots, \tau_M, a_1, \ldots, a_M)$. $\square$

Supplementary Explanations

Probabilities and profits pertaining to stockouts

Lost Sales: Given stockout probabilities ν_1, ν_2, retailers can find equilibrium prices p^1, p^2 as detailed below and then implement the commonly used (Q, R) inventory policy: retailer i orders fixed quantity Q_i when its inventory level reaches the reorder point R_i. To assess the fill rate, we can assume that the total demand arriving to retailers has a Poisson distribution with rate 1, which can always be achieved by appropriately scaling time. Then retailer i experiences Poisson demand with rate $\rho^i_{ls}(p^1, p^2)/(1-\nu_i)$ and its demand during lead time LT_i is Poisson with rate $\rho^i_{ls}(p^1, p^2)LT_i/(1 - \nu_i)$. Denoting this demand by $\mathcal{P}(\rho^i_{ls}(p^1, p^2)LT_i/(1 - \nu_i))$, we have the expected stockouts $\mathrm{E}(\mathcal{P}(\rho^i_{ls}(p^1, p^2)LT_i/(1 - \nu_i)) - R_i)^+$, which should be equal to $\nu_i Q_i/(1 - \nu_i)$ for a fill rate of $1 - \nu_i$. Hence, R_i satisfies $\mathrm{E}(\mathcal{P}(\rho^i_{ls}(p^1, p^2)LT_i/(1 - \nu_i)) - R_i)^+ = \nu_i Q_i/(1 - \nu_i)$. Note that this fill rate equation does not distinguish between customers preferring retailer i and the others. This equation can be used to calculate R_i for given prices (p^1, p^2) and stockout probabilities ν_1, ν_2. Essentially we can replace R's with ν's as policy parameters and speak of (Q, ν) policy. This approach of basing policy parameters on fill rates is especially useful when fill rates are used to benchmark inventory performance among firms and when the stockout costs, being intangible, are difficult to estimate making an approach based on cost minimization unviable.

In the (Q, ν) model, retailer i sells Q_i units in each inventory cycle. Since the fill rate is $(1 - \nu_i)$, stocked-out demand in a cycle is $\nu_i Q_i/(1 - \nu_i)$ and the total demand is $Q_i/(1 - \nu_i)$.

The expected length of time between two sales in a row at retailer i is $1/\rho^i_{ls}(p^1,p^2)$. An inventory cycle lasts over Q_i sales, which in terms of time is $Q_i/\rho^i_{ls}(p^1,p^2)$. The cycle length is $Q_i/\rho^i_{ls}(p^1,p^2)$ and the profit over a cycle is $(p^i-c_i)Q_i$. Using renewal theory, we obtain the profit per unit time.

$$\text{Profit of Retailer } i \text{ under lost sales}: \quad \frac{(p^i-c_i)Q_i}{Q_i/\rho^i_{ls}(p^1,p^2)} = (p^i-c_i)\rho^i_{ls}(p^1,p^2).$$

Interestingly, this profit does not depend on the quantity Q_i. In other words, maximizing the profit per unit time is equivalent to maximizing the profit made from each customer demand. A main driver of this result is that retailer i using (Q,ν) policy sells exactly Q_i units in a cycle.

Retailer Favoring Customers in a Duopolistic Market: With the demand process as above, the demand during lead time at retailer i is Poisson with rate $LT_i(\rho^{i,n}_{ret}(p^1,p^2)/(1-\nu_i)) = LT_i(\rho^{i,b}_{ret}(p^1,p^2)/\nu_i)$ and the expected stockouts is $\mathrm{E}(\mathcal{P}(LT_i(\rho^{i,n}_{ret}(p^1,p^2)/(1-\nu_i)))-R_i)^+$. Hence, R_i can be found from $\mathrm{E}(\mathcal{P}(LT_i(\rho^{i,n}_{ret}(p^1,p^2)/(1-\nu_i)))-R_i)^+ = \nu_i Q_i$ for given prices and fill rates. Replacing R with ν, we arrive again at a (Q,ν) policy.

In the (Q,ν) policy, the total demand at retailer i in a cycle is Q_i and this cycle has a length of $Q_i/(\rho^{i,n}_{ret}(p^1,p^2)+\rho^{i,b}_{ret}(p^1,p^2))$. During the cycle $(1-\nu_i)Q_i$ units are sold from inventory and each yield a profit of p^i-c_i while $\nu_i Q_i$ units are backordered and each yield a profit of $p^i-c_i-d_i$. So the total profits is $(p^i-c_i-\nu_i d_i)Q_i$. Then the profit per time of retailer i from retailver favoring customers is:

$$\begin{aligned}\Pi^i_{ret}(p^1,p^2) &= \frac{(p^i-c_i-\nu_i d_i)Q_i}{Q_i/(\rho^{i,n}_{ret}(p^1,p^2)+\rho^{i,b}_{ret}(p^1,p^2))} \\ &= (p^i-c_i-\nu_i d_i)(1-W^i(p^i))[\phi_i+\phi_{-i}W^{-i}(p^{-i})].\end{aligned}$$

Once more, the profit is independent of the order quantity and the profit per time coincides with the profit per customer.

Availability Favoring Customers in a Duopolistic Market: The sales probabilities $\rho^{i,n}_{ava}, \rho^{i,b}_{ava}$ for availability favoring customers are based on Figure A.2. The availability favoring profit Π^i_{ava} is analogous to retailer favoring profits detailed above.

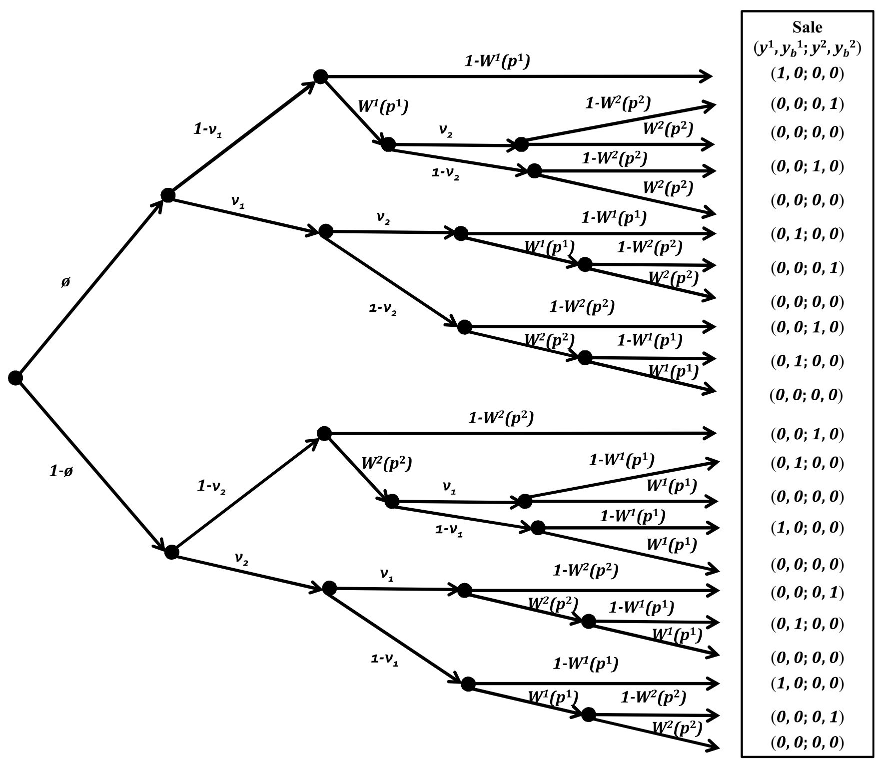


Figure A.2. An availability favoring customer's decision tree from the firm's perspective.

Visual examination for uniform WTPs, Number of parameters and WTP distributions

When the WTP has a □ distribution for a product, its sales drops linearly with the price. We plot (price, sales) pairs for the candy melts, and visually check for the uniform distribution of WTPs. The weekly sales data are separated into four non-overlapping seasons. To reduce the effect of the price of the other product when plotting the (price, sales) pairs for a product, we keep the other price stable. This is achieved by defining three price ranges for the other product (low, medium and high) and plotting three graphs for each product and each season. With 2 products, 4 seasons and 3 price ranges, we end up with 24 plots. Nineteen of these

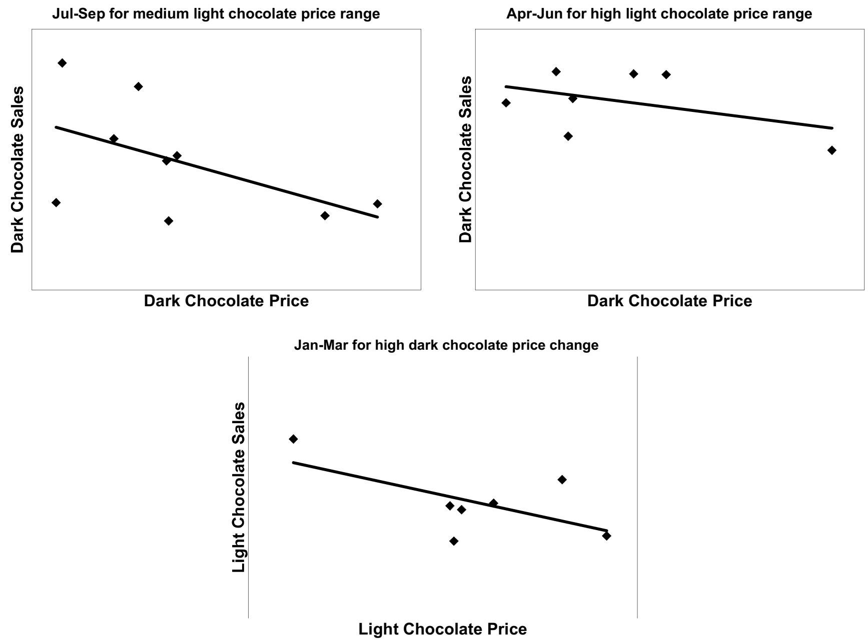


Figure A.3. Sales vs. price graphs: Evidence for uniform distribution of WTPs.

plots support $\Box$ distributed WTPs, and three of these are shown in Figure A.3. So, $\Box$ distribution appears to be reasonable for the WTPs, at least for the candy melt data.

WTP-choice model with $(\Box, \delta = 1)$, has five parameters a_1, a_2, b_1, b_2, and ϕ to estimate. To avoid a concern of overfitting, we restrict the number of parameters (degrees of freedom) in the model to three –the same number as in Logit model. Then we increase the number of parameters to four to observe the robustness of the L_{wtp} values with respect to the number of parameter. Table A.1 shows the L_{wtp} values for four sets of estimated parameters: $\{b_1, b_2, \phi\}$, $\{a_1, a_2, \phi\}$, $\{a_2, b_1, b_2, \phi\}$, $\{a_1, a_2, b_2, \phi\}$. The third and fourth sets are obtained by respectively appending a_2 and b_2 to the first and second sets. In Table A.1, the L_{wtp} improves as we increase the number of estimated parameters. These improvements are sufficient to beat the L_{sl} with $(\Box, \delta = 1)$ in the cases of ketchup and tuna.

To investigate the effect of WTP distribution and δ, we consider two more WTP-choice models: $(\triangle, \delta = 1)$ and $(\llcorner, \delta \leq 1)$. Adding these to WTP-choice model of $(\triangle, \delta \leq 1)$ in Table 1.5 and $(\Box, \delta = 1)$ in Table 1.6, we have four models. Table A.2 reports the differences

Table A.1. Log-likelihoods for different set of estimated parameters for ($\Box, \delta = 1$) WTP-choice model.

	L_{wtp} when				L_{sl}
	three parameters estimated		four parameters estimated		
	b_1,b_2,ϕ	a_1,a_2,ϕ	a_2,b_1,b_2,ϕ	a_1,a_2,b_2,ϕ	
Yogurt	-1,853	-2,297	-1,844	-2,074	-1,835
Ketchup	-4,560	-4,264	-3,755	-4,281	-4,169
Candy melt	-11,634	-16,092	-11,561	-12,794	-11,498
Tuna	-13,157	-11,870	-11,068	-11,742	-11,143

between these L values. There switching from ($\triangle, \delta \leq 1$) to ($\triangle, \delta = 1$) in the first column changes the L values by $(L_{wtp}(\triangle, \delta = 1) - L_{wtp}(\triangle, \delta \leq 1))/|L_{wtp}(\triangle, \delta \leq 1)| = L_{wtp}(\triangle, \delta = 1)/|L_{wtp}(\triangle, \delta \leq 1)| + 1$, other columns have the same interpretation.

Table A.2. Differences in log-likelihoods as switching from one WTP-choice models to another.

	From ($\triangle, \delta \leq 1$) to ($\triangle, \delta = 1$) $\frac{L_{wtp}(\triangle, \delta = 1)}{\lvert L_{wtp}(\triangle, \delta \leq 1)\rvert} + 1$ in %	From ($\Box, \delta = 1$) to ($\triangle, \delta = 1$) $\frac{(L_{wtp}(\triangle, \delta = 1)}{\lvert L_{wtp}(\Box, \delta = 1)\rvert} + 1$ in %	From ($\triangle, \delta \leq 1$) to ($\llcorner, \delta \leq 1$) $\frac{(L_{wtp}(\llcorner, \delta \leq 1)}{\lvert L_{wtp}(\triangle, \delta \leq 1)\rvert} + 1$ in %
Yogurt	-0.85	0.63	-0.33
Ketchup	-29.10	8.68	0.05
Candy melt	-2.32	0.78	0.47
Tuna	-5.53	15.72	-1.23

Most changes in Table A.2 occur in the ketchup and tuna data. For example, the L value decreases by 29.10% when $\delta = 1$ as opposed to $\delta \leq 1$ in the ketchup data, so quite a few ketchup customers are not interested in the ketchup captured by the data. The last column shows the difference in L_{wtp} values between $\llcorner$ and $\triangle$ WTPs, the two models are slightly different in terms of L_{wtp} values. The maximum difference is with tuna data where $\triangle$ WTP fits better.

APPENDIX B

PROOFS FOR CHAPTER 2

Proof of Proposition 1: The derivatives of Q_1^{1*} with respect to α, c_1, c_2 and c_3, respectively, are $\frac{\partial Q_1^{1*}}{\partial \alpha} = \frac{a - 2c_1 - 2c_2 + 3c_3}{2(\alpha+2)^2} > 0$, $\frac{\partial Q_1^{1*}}{\partial c_1} = -\frac{\alpha+1}{\alpha+2} < 0$, $\frac{\partial Q_1^{1*}}{\partial c_2} = \frac{1}{(\alpha+2)} > 0$, and $\frac{\partial Q_1^{1*}}{\partial c_3} = -\frac{1-\alpha}{\alpha+2} < 0$. Similarly, we can prove the results for Q_2^{1*} and Q_e^{1*}. □

Proof of Proposition 2: $\frac{\partial E(S^1)}{\partial \alpha} = \frac{-a - (\alpha^2+4\alpha+2)c_1 + 2c_2 + (\alpha^2+4\alpha+1)c_3}{2(\alpha+2)^2}$. So, if $2c_2 + (\alpha^2+4\alpha+1)c_3 > a + (\alpha^2+4\alpha+2)c_1$, we have $\frac{\partial E(S^1)}{\partial \alpha} > 0$. It is easy to verify that $\frac{\partial E(S^1)}{\partial c_1} < 0$, $\frac{\partial E(S^1)}{\partial c_2} < 0$, and $\frac{\partial E(S^1)}{\partial c_3} < 0$. □

Proof of Proposition 3:

$$\begin{aligned}\frac{\partial E(S^2)}{\partial \alpha} = {} & \frac{1}{(3\alpha+4+\alpha^2)^2}(6c_2 - c_3 + a(-1+\alpha(2+\alpha)) - c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha))) \\ & +\alpha(4c_2 + c_3(10+\alpha(16+\alpha(6+\alpha))))).\end{aligned}$$

If $6c_2 - c_3 + a(-1+\alpha(2+\alpha)) - c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha))) + \alpha(4c_2 + c_3(10+\alpha(16+\alpha(6+\alpha)))) > 0$, then $\frac{\partial E(S^2)}{\partial \alpha} > 0$. It is easy to verify that $\frac{\partial E(S^2)}{\partial c_1} < 0, \frac{\partial E(S^2)}{\partial c_2} < 0$ and $\frac{\partial E(S^2)}{\partial c_3} < 0$. □

Proof of Proposition 4: $\frac{\partial E(S^3)}{\partial \alpha} = \frac{c_3 - c_1}{2} > 0$. Therefore, the expected total market output $E(S^3)$ is increasing in α. Similarly, we can prove the other results for $E(S^3)$ and $E(p^3)$. □

Proof of Proposition 5: $\frac{\partial E(S^4)}{\partial \alpha} = \frac{a - 6c_1 + c_2 + 4c_3}{12} > 0$. Therefore, the expected total market output $E(S^3)$ is increasing in α. Similarly, we can prove other results for $E(S^4)$ and $E(p^4)$. □

Proof of Proposition 6: Proposition 6 is easily derived from the first derivative of $E(S^5)$ and $E(p^5)$ with respect to α, c_1, c_2, and c_3, respectively. □

Proof of Proposition 7: We can prove this result by taking the first derivatives of $E(S^6)$ and $E(p^6)$ with respect to α, c_1, c_2, and c_3, respectively.

Proof of Proposition 8: By straightforward comparison of the profit expressions obtained in section 2.3x. $\square$

Proof of Proposition 9: We have from the expected profits calculated before and $c_2 = c_3$ that $E(\Pi_C^5) > \alpha\frac{(a-c_2)^2}{8} + (1-\alpha)\frac{(a-c_2)^2}{16}$ and $E(\Pi_S^5) < \alpha\frac{(a-c_2)^2}{16} + (1-\alpha)\frac{(a-c_2)^2}{8}$. Substituting $\alpha = 0.5$ in both inequalities we have $E(\Pi_C^5) > \frac{3(a-c_2)^2}{32}$ and $E(\Pi_S^5) < \frac{3(a-c_2)^2}{32}$. Therefore, $E(\Pi_C^5) > E(\Pi_S^5)$, which completes the proposition. This means that when Supplier U is sufficiently reliable, i.e., $\alpha \geq 0.5$, C has a higher expected profit than S. $\square$

Proof of Proposition 10: Let the capacity reserved by C be $K^*(c_r)$, and the emergency order quantity at c_r be $Q_e^{k*}(c_r)$. Correspondingly, there are three cases – (a) $K^*(c_r) > Q_e^{k*}(c_r)$; (b) $K^*(c_r) < Q_e^{k*}(c_r)$; and (c) $K^*(c_r) = Q_e^{k*}(c_r)$. In Case (a); the reserved capacity is higher than the emergency order quantity. C saves $c_r(K^*(c_r) - Q_e^{k*}(c_r))$ by reserving a capacity of $Q_e^{k*}(c_r)$. Therefore, it is not optimal for C to choose $K^*(c_r) > Q_e^{k*}(c_r)$. On the other hand, in Case (b) the reserved capacity $K^*(c_r)$ is less than $Q_e^{k*}(c_r)$. Therefore, C cannot fulfill the optimal emergency order. Therefore, it is not optimal for C to choose $K^*(c_r) < Q_e^{k*}(c_r)$. Therefore, $K^*(c_r) = Q_e^{k*}(c_r)$ is optimal for C. From section 2.3.1, we see that Q_e^{k*} decreases in c_3^r and since c_r is a sunk cost to decide Q_e^{k*}. $\square$

APPENDIX C

PROOFS FOR CHAPTER 3

Proof for Proposition 11

Proof. a) Let $\mathcal{S}^*_{ppu} \neq \phi$ and $\mathcal{S}^*_{sub} \neq \phi$ be the set of products chosen by a consumer that maximizes her utility. The total surplus of the consumer is

$$\tilde{U}(\mathcal{S}^*_{ppu}, \mathcal{S}^*_{sub}) = \sum_{k \in \mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub}} z(k) - \mathbb{I}_{|\mathcal{S}^*_{sub}|} p_{sub} - |\mathcal{S}^*_{ppu}| p_{ppu}, \tag{C.1}$$

when consumer consumes $|\mathcal{S}^*_{ppu}|$ products on pay-per-unit distributor at a price p_{ppu} for each product. she can instead consume all products – $|\mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub}|$ on subscription distributor that increases her surplus from consuming products by $\mathcal{S}^*_{ppu} p_{ppu}$. Accordingly, $\tilde{U}(\mathcal{S}^*_{ppu}, \mathcal{S}^*_{sub}) < \tilde{U}(\phi, \mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub})$. This means $\mathcal{S}^*_{ppu} \neq \phi$ and $\mathcal{S}^*_{sub} \neq \phi$ is not optimal. However, if $p^{sub} \geq |\mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub}| p_{ppu}$, it is better for consumer to choose pay-per-unit distributor only, i.e. $\tilde{U}(\mathcal{S}^*_{ppu}, \mathcal{S}^*_{sub}) < \tilde{U}(\phi, \mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub}) < \tilde{U}(\mathcal{S}^*_{ppu} \bigcup \mathcal{S}^*_{sub}, \phi)$.

b) We know that $\mathcal{S}^*_{ppu} \neq \phi$ and $i \in \mathcal{S}^*_{ppu}$. Let $j < i \in \mathcal{M}$ and $j \notin \mathcal{S}^*_{ppu}$ and $\tilde{U}(\mathcal{S}^*_{ppu}, \mathcal{S}^*_{sub})$ be the utility of a consumer. If she chooses product j instead of i from $\mathcal{M}$, then her consumption set is $\tilde{\mathcal{S}}_{ppu}$, and her surplus is $\tilde{U}(\tilde{\mathcal{S}}_{ppu} \mathcal{S}^*_{sub})$. Since, $z(i) \leq z(j)$ her surplus increases by consuming j instead of i, i.e., $\tilde{U}(\tilde{\mathcal{S}}^*_{ppu}, \mathcal{S}^*_{sub}) - \tilde{U}(\mathcal{S}^*_{ppu}, \mathcal{S}^*_{sub}) = z(j) - z(i) \geq 0$. Therefore, $\mathcal{S}^*_{ppu}$ does not maximizes her surplus and her surplus is more with $\tilde{\mathcal{S}}_{ppu}$. Subsequently, if $i \in \mathcal{S}^*_{ppu}$ then for all $j < i \in \mathcal{M}$, we have $j \in \mathcal{S}^*_{ppu}$. Consumers choose products with higher net utilities in their optimal consumption sets. The proof follows similarly if the consumers instead choose subscription pricing.

c) We know from a) that if consumers choose subscription pricing then $p_{sub} < |\mathcal{S}^*_{sub}| p_{ppu}$. Choosing subscription pricing but consuming $\mathcal{S}^*_{ppu}$ is feasible and yields a higher surplus.

Therefore if a pay-per-unit consumer switches to subscription pricing and pays p_{sub} then she increases her utility by consuming more, i.e., $p_{sub} > p_{ppu}$ for $p_{sub} < |\mathcal{S}^*_{sub}|p_{ppu}$.

□

Proof of Propositions 12, 13, and 14

Proof. A straightforward comparison of monopolist profit under subscription only vis- à -vis pay-per-unit only shows that subscription always yields a higher profit. A monopolist who can sell by both subscription and pay-per-unit channels can always choose to sell only by subscription alone and recover the same profit as the optimal profit of a subscription pricing based monopolist.

When a monopolist sells through both subscription and pay-per-unit pricing modalities, the demands and the profits via each channel are:

$$N_{sub}(p_{sub}; p_{ppu}) = N \int_{a_1}^{a_2} \frac{1}{a_2 - a_1} \mathbb{I}_{\{a \geq \tilde{a}(p_{sub}, p_{ppu})\}} da = \frac{N(a_2 - \tilde{a}(p_{sub}, p_{ppu}))}{a_2 - a_1}$$

and

$$D_{ppu}(p_{ppu}; p_{sub}) = N \int_{a_1}^{a_2} \frac{1}{a_2 - a_1} \mathbb{I}_{\{a < \tilde{a}(p_{sub}, p_{ppu})\}} \frac{a^2}{4(c + p_{ppu})^2} da = \frac{N(\tilde{a}(p_{sub}, p_{ppu})^3 - a_1^3)}{12(c + p_{ppu})^2(a_2 - a_1)}.$$

Subsequently, the total profit of the monopolist is

$$\Pi^{mon}_{dual}(p_{sub}, p_{ppu}) = N_{sub}(p_{sub}; p_{ppu})p_{sub} + D_{ppu}(p_{sub}; p_{ppu})p_{ppu}.$$

The corresponding profit maximization problem for the monopolist is $\max_{\{p_{sub}, p_{ppu}\}} \Pi^{mon}_{dual}(p_{sub}, p_{ppu})$. From the first order condition for p_{sub}, we have

$$p^*_{sub}(p_{ppu}) = \frac{a_2^2 p_{ppu}(c + p_{ppu})}{c(2c + 3p_{ppu})^2}, \tag{C.2}$$

and we also see that $\Pi^{mon}_{dual}(p_{sub}, p_{ppu})$ is concave in p_{sub} as the second derivative is negative. Therefore, for a given p_{ppu} there is a unique p^*_{sub} as given by (C.2). Therefore, we can simplify the monopolist's maximization problem to a single variable problem in just p_{ppu} as $\max_{p_{ppu}\}} \Pi^{mon}_{dual}(p^*_{sub}(p_{ppu}), p_{ppu})$. Now we show that $\Pi^{mon}_{dual}(p^*_{sub}(p_{ppu}), p_{ppu})$ is increasing in p_{ppu}. Taking the first derivative of $\Pi^{mon}_{dual}(p^*_{sub}(p_{ppu}), p_{ppu})$ w.r.t. p_{ppu} we see that it is always positive. Therefore setting $p^*ppu = \infty$ is optimal for a optimal. This means, that the monopolist must use only subscription channel to sell to consumers even if he has the ability to sell through both pay-per-unit and subscription channels. □

Best response of distributors: Best response for pay-per-unit distributor:

$$p^*_{sub}(p_{ppu}, K) \quad = \quad \begin{cases} \dfrac{a_1^2 p_{ppu}}{4c(c+p_{ppu})} & \text{if } 2a_2 < 3a_1, \\ \dfrac{a_2^2 p_{ppu}}{9c(c+p_{ppu})} & \text{if } 3a_1 \leq 2a_2 \leq 3a_2 \ . \end{cases} \tag{C.3}$$

The first order condition for $a_2 \geq \tilde{a}(p_{sub}, p^*_{ppu}(p_{sub}, w)) \geq a_1$ gives the best response for pay-per-unit distributor $p^*_{ppu}(p_{sub}, w)$ that must satisfy:

$$a_1^3 p_{ppu}(c - p_{ppu} + 2w) + 4\left(2p_{ppu}(p_{ppu} - 2w) + c(p_{ppu} - 3w)\right)\left(\frac{cp_{sub}(c+p_{ppu})}{p_{ppu}}\right)^{\frac{3}{2}} = 0. \tag{C.4}$$

The best response for the pay-per-unit distributor when $\tilde{a}(p_{sub}, p^*_{ppu}(p_{sub}, w)) > a_2$ is

$$p^*_{ppu}(p_{sub}, w) = c + 2w \text{ if } p_{sub} > \frac{a_2^2(c+2w)}{16(c+w)^2}. \tag{C.5}$$

Proof of Theorem 5

Proof. (a) We split the proof in two parts: (a) $2a_2 < 3a_1$; and (b) $2a_2 \geq 3a_1$.

Low Heterogeneity $2a_2 < 3a_1$**:** $(p^e_{ppu}, p^e_{sub}) = \left(w, \dfrac{a_1^2 w}{4c(c+w)}\right)$ is the Nash equilibrium.

If, $(p^e_{ppu}, p^e_{sub}) = \left(w, \dfrac{a_1^2 w}{4c(c+w)}\right)$ is a Nash equilibrium, then we must have

$$\Pi_{ppu}\left(w, \frac{a_1^2 w}{4c(c+w)}\right) = 0 \geq \Pi_{ppu}\left(p, \frac{a_1^2 w}{4c(c+w)}\right). \tag{C.6}$$

For $p < w$, $\Pi_{ppu}\left(p, \dfrac{a_1^2 w}{4c(c+w)}\right) < 0$ and (C.6) is satisfied. On the other hand, When $p > w$, we know from previous results that $\dfrac{\partial \tilde{a}(p_{ppu}, p_{sub})}{\partial p_{ppu}} < 0$, therefore demand for pay-per-unit distributor cannot be more than 0. Therefore, it is not in the best interest for pay-per-unit distributor to charge more than w. Again (C.6) holds. Let us now check if this equilibrium holds for subscription distributor

$$\Pi_{sub}\left(w, \frac{a_1^2 w}{4c(c+w)}\right) \geq \Pi_{sub}\left(w, p\right).$$

This holds, because for a given p_{ppu} and $2a_2 < 3a_1$, subscription distributor's best response is $\frac{a_1^2 p_{ppu}}{4c(c+p_{ppu})}$. Therefore, $(p^e_{ppu}, p^e_{sub}) = \left(w, \frac{a_1^2 w}{4c(c+w)}\right)$ is a Nash equilibrium for $2a_2 < 3a_1$.

Now we show that $(p^e_{ppu}, p^e_{sub}) = \left(w, \frac{a_1^2 w}{4c(c+w)}\right)$ is a unique Nash equilibrium for $2a_2 < 3a_1$. Let there is another Nash equilibrium $(p^e_{ppu}, p^e_{sub}) = \left(\bar{p}, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right)$ for $2a_2 < 3a_1$ and $\bar{p} \neq w$. It is straightforward to see that $\bar{p} < w$ can not be an equilibrium price for pay-per-unit distributor because this gives non-positive profit, and she can set $\bar{p} = w$ to do better. Let us now assume $p > w$ gives us Nash equilibrium. Therefore,

$$\Pi_{ppu}\left(\bar{p}, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right) = 0 \geq \Pi_{ppu}\left(p, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right). \quad \text{(C.7)}$$

Let us choose $p = \bar{p} - \epsilon$ for some $\epsilon > 0$. Now, pay-per-unit distributor is better off as $\tilde{a}\left(p, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right) > \tilde{a}\left(\bar{p}, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right) = a_1$. Therefore, there is a non-zero demand for $Ret_p pu$, as $a_2 > \tilde{a}\left(p, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right) > a_1$, which means

$$\Pi_{ppu}\left(\bar{p}, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right) = 0 < \Pi_{ppu}\left(p, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right).$$

This is a contradiction if $(p^e_{ppu}, p^e_{sub}) = \left(\bar{p}, \frac{a_1^2 \bar{p}}{4c(c+\bar{p})}\right)$ is an equilibrium for $p > w$. This proves the uniqueness of $(p^e_{ppu}, p^e_{sub}) = \left(w, \frac{a_1^2 w}{4c(c+w)}\right)$ as a Nash equilibrium.

High Heterogeneity $2a_2 \geq 3a_1$: Solving the best response equations simultaneously gives us the unique Nash equilibrium as [1]

$$(p^e_{sub}, p^e_{ppu}) = \left(\frac{a_2^2 \left(\dfrac{-B+\sqrt{B^2-4AC}}{2A} \right)}{9c\left(c + \dfrac{-B+\sqrt{B^2-4AC}}{2A} \right)}, \frac{-B+\sqrt{B^2-4AC}}{2A} \right), \tag{C.8}$$

where $A = 8a_2^3 - 27a_1^3, B = 27a_1^3\{c+2w\} + 4a_2^3\{c-4w\}$ and $C = -12cwa_2^3$. $A > 0$ because $2a_2 > 3a_1$ and $\tilde{a}(p^e_{ppu}, p^e_{sub}) = 2a_2/3$.

(b) Given the equilibrium prices (p^e_{ppu}, p^e_{sub}), it is easy to see that $\lim_{w\to 0}(p^e_{ppu}, p^e_{sub}) = (0,0)$. This implies that the distributors engage in a price war when the marginal costs to the distributors are zero. However, when $w > 0$, we see that the distributors have $p^e_{ppu} > 0$ and $p^e_{sub} > 0$. Therefore, for $w > 0$ there is always a stable equilibrium in prices.

(c) We know when $2a_2 < 3a_1$, equilibrium prices are $(p^e_{ppu}, p^e_{sub}) = \left(w, \dfrac{a_1^2 w}{4c(c+w)} \right)$, where the market share of subscription distributor is 1. When $2a_2 < 3a_1$, the equilibrium prices are derived in (i), we find $\tilde{a}(p^e_{ppu}, p^e_{sub}) = \dfrac{a_2}{3(a_2-a_1)}$. Hence, the market share of subscription distributor is $\min\left\{1, \dfrac{a_2}{3(a_2-a_1)}\right\}$, which is surprisingly equal to the monopoly case. □

Proposition 19. *i) The price charged by pay-per-unit and subscription based distributors increases in the wholesale price w and is independent of the licensing fee K.*

ii) When the wholesale price w is very high, the equilibrium pay-per-unit distributor price is very high, her equilibrium demand is zero and the subscription distributor charges the monopoly subscription price.

iii) When the wholesale price w is very high, the equilibrium demand for the pay-per-unit distributor is zero.

Proof of Proposition 19

[1] $(p^e_{sub}, p^e_{ppu}) = (0,0)$ is a solution to (C.4), however this is not a feasible Nash equilibrium as $p^e_{ppu} \geq w$, when $w = 0$ the solution is valid and indicates price wars.

Proof. i) For $2a_2 \geq 3a_1$, we have

$$\frac{\partial p^e_{ppu}(w)}{\partial w} = \frac{(8a_2^3 - 27a_1^3)(c+2w) + \sqrt{48a_2^3\left(-27a_1^3+8a_2^3\right)cw + \left(4a_2^3(c-4w)+27a_1^3(c+2w)\right)^2}}{\sqrt{48a_2^3\left(-27a_1^3+8a_2^3\right)cw + \left(4a_2^3(c-4w)+27a_1^3(c+2w)\right)^2}}$$
$$> 1,$$

and for $2a_2 < 3a_1$, we have $\frac{\partial p^e_{ppu}(w)}{\partial w} = 1$. This shows that the equilibrium price $p^e_{ppu}(w)$ is increasing in w. We show the same is valid for $p^e_{sub}(w)$.

$$\frac{\partial p^e_{sub}(w)}{\partial w} = \max\left\{\frac{a_2^2}{9(c+p^e_{ppu}(w))^2}, \frac{a_1^2}{4(c+p^e_{ppu}(w))^2}\right\}\frac{\partial p^e_{ppu}(w)}{\partial w} \geq 0.$$

This shows that the equilibrium price $p^e_{sub}(w)$ is increasing in the wholesale price w.

ii) Now we show the asymptotic properties of equilibrium prices as $\lim_{w\to\infty}$. We know that

$$\frac{\partial p^e_{ppu}(w)}{\partial w} \geq 1 \;\Rightarrow\; \lim_{w\to\infty} p^e_{ppu}(w) = \infty,$$

$$\lim_{w\to\infty} p^e_{sub}(w) = \lim_{w\to\infty} \max\left\{\frac{a_2^2}{9c(1+c/p^e_{ppu}(w))}, \frac{a_1^2}{4c(1+c/p^e_{ppu}(w))}\right\} = \max\left\{\frac{a_2^2}{9c}, \frac{a_1^2}{4c}\right\}.$$

The second order conditions on $p^e_{ppu}(w)$ for $2a_2 > 3a_1$ is

$$\frac{\partial^2 p^e_{ppu}(w)}{\partial w^2} = -\frac{48a_2^3\left(729a_1^6 - 270a_1^3a_2^3 + 16a_2^6\right)c^2}{\left(48a_2^3\left(-27a_1^3+8a_2^3\right)cw + \left(4a_2^3(c-4w)+27a_1^3(c+2w)\right)^2\right)^{3/2}},$$

and the sign of the above expression depends exclusively on the sign of $(729a_1^6 - 270a_1^3a_2^3 + 16a_2^6)$. Therefore the above expression is positive for $2a_2 > 3a_1 > \sqrt[3]{2}a_2$ and negative for $\sqrt[3]{2}a_2 > 3a_1$. When $\sqrt[3]{2}a_2 = 3a_1$, $\frac{\partial^2 p^e_{ppu}(w)}{\partial w^2} = 0$.

From this we can derive the limiting demands for pay-per-unit and subscription based distributor as

$$\lim_{w\to\infty} N_{sub}(p^e_{ppu}, p^e_{sub}) = \frac{Na_2}{3(a_2-a_1)} = N^{mon}_{sub}, \; \lim_{w\to\infty} D_{ppu}(p^e_{ppu}, p^e_{sub}) = 0, \tag{C.9}$$

and the corresponding profits are

$$\lim_{w\to\infty} \Pi_{sub}(p^e_{ppu}, p^e_{sub}) = \frac{Na_2^3}{27c(a_2-a_1)} = \Pi^{mon}_{sub}, \; \lim_{w\to\infty} \Pi_{ppu}(p^e_{ppu}, p^e_{sub}) = 0. \tag{C.10}$$

□

Proof of Proposition 15

Proof. The objective for the content provider is:

$$
\begin{aligned}
& max_w[p^e_{sub}D_{sub}(p^e_{ppu},p^e_{sub}) + wD_{ppu}(p^e_{ppu},p^e_{sub})] \\
= \; & max_w \begin{cases} N\dfrac{a_1^2 w}{4c(c+w)} & \text{if } 2a_2 < 3a_1, \\ \dfrac{Na_2 p^e_{sub}(w)}{3(a_2-a_1)} + \dfrac{Nw(8a_2^3-27a_1^3)}{324(a_2-a_1)(c+p^e_{ppu}(w))^2} & \text{if } 2a_2 \geq 3a_1. \end{cases}
\end{aligned} \tag{C.11}
$$

Let us first show the result for $2a_2 < 3a_1$, and use the objective for content provider as given in (C.11). The result follows from Proposition 19. Next, we show the result is valid even for $2a_2 \geq 3a_1$. We need to show that profit of the content provider is increasing in w. To demonstrate this, we take the partial derivative of content provider's profit function with respect to w and obtain:

$$
\begin{aligned}
& \frac{\partial p^e_{sub}(w)}{\partial w}D_{sub}(p^e_{ppu},p^e_{sub}) + \frac{\partial D_{sub}(p^e_{ppu},p^e_{sub})}{\partial w}p^e_{sub}(w) + D_{ppu}(p^e_{ppu},p^e_{sub}) + w\frac{\partial D_{ppu}(p^e_{ppu},p^e_{sub})}{\partial w} \\
= \; & \frac{a_2}{3}\frac{\partial p^e_{sub}(w)}{\partial w} + \frac{(8a_2^3-27a_1^3)}{108(c+p^e_{ppu}(w))^3}\left[c+p^e_{ppu}(w) - 2w\frac{\partial p^e_{ppu}(w)}{\partial w}\right] \\
= \; & \frac{4a_2^3(c+p^e_{ppu}(w))}{108(c+p^e_{ppu}(w))^3}\frac{\partial p^e_{ppu}(w)}{\partial w} + \frac{(8a_2^3-27a_1^3)}{108(c+p^e_{ppu}(w))^3}\left[c+p^e_{ppu}(w) - 2w\frac{\partial p^e_{ppu}(w)}{\partial w}\right] \\
= \; & \frac{4a_2^3(c+p^e_{ppu}(w))\frac{\partial p^e_{ppu}(w)}{\partial w} + (8a_2^3-27a_1^3)\left[c+p^e_{ppu}(w) - 2w\frac{\partial p^e_{ppu}(w)}{\partial w}\right]}{108(c+p^e_{ppu}(w))^3} \\
= \; & \lim_{w\to\infty}\frac{4a_2^3(c+p^e_{ppu}(w))\frac{\partial p^e_{ppu}(w)}{\partial w} + (8a_2^3-27a_1^3)\left[c+p^e_{ppu}(w) - 2w\frac{\partial p^e_{ppu}(w)}{\partial w}\right]}{108(c+p^e_{ppu}(w))^3} > 0.
\end{aligned}
$$

The last statement is true if there is a $\tilde{w}$ s.t. $\tilde{w} = \dfrac{c+p^e_{ppu}(\tilde{w})}{2\frac{\partial p^e_{ppu}(\tilde{w})}{\partial w}}$. $\tilde{w}$ satisfies $w = \dfrac{c+p^e_{ppu}(w)}{2\frac{\partial p^e_{ppu}(w)}{\partial w}}$. Left hand side in the equation is increasing with unit slope. Derivative of right hand side with w gives $1 - \dfrac{c+p^e_{ppu}(w)}{\left(\frac{\partial p^e_{ppu}(w)}{\partial w}\right)^2}\dfrac{\partial^2 p^e_{ppu}(w)}{\partial w^2} > 1$. This shows that both left hand side and the right hand side are increasing in w, moreover we know that there exists a solution $\tilde{w}$ that can be found numerically. The solution is unique since the right hand side is increasing faster than the left hand side. Therefore, $w > \tilde{w} \Rightarrow c + p^e_{ppu}(\tilde{w}) > 2\tilde{w}\frac{\partial p^e_{ppu}(\tilde{w})}{\partial \tilde{w}}$. So the maximum is at $w^e_1 = \arg\max_{w \geq 0} \Pi_{CP}(w) = \infty$.

□

Proof of Proposition 16

Proof. The content provider decides w^e_2 such that

$$w^e_2 = \arg\max_{w \geq 0} wD_{ppu}(p^e_{ppu}, p^e_{sub}) = \arg\max_{w \geq 0} \frac{Nw(8a_2^3 - 27a_1^3)}{324(a_2 - a_1)(c + p^e_{ppu}(w))^2}.$$

$$\lim_{w \to 0} wD_{ppu}(p^e_{ppu}, p^e_{sub}) = 0 \text{ and } \lim_{w \to \infty} wD_{ppu}(p^e_{ppu}, p^e_{sub}) = 0.$$

For some $w > 0$ we have $D_{ppu}(p^e_{ppu}, p^e_{sub}) > 0$ and $wD_{ppu}(p^e_{ppu}, p^e_{sub}) > 0$. Therefore, there exists some $w^e_2 \in (0, \infty)$ that maximizes the profit of the content provider. From the first order condition, w^e_2 satisfies

$$w = \frac{c + p^e_{ppu}(w)}{2\frac{\partial p^e_{ppu}(w)}{\partial w}}. \tag{C.12}$$

Unique solution: Left hand side in above equation is increasing with a slope 1. Taking derivative of the right-hand side w.r.t. w, we get

$$1 - \frac{c + p^e_{ppu}(w)}{\left(\frac{\partial p^e_{ppu}(w)}{\partial w}\right)^2} \frac{\partial^2 p^e_{ppu}(w)}{\partial w^2} > 1 \text{ for } \sqrt[3]{2}a_2 > 3a_1. \tag{C.13}$$

This shows that both left hand side and right hand side are increasing in w. The solution to (C.12) is unique as the right hand side is increasing faster than the left hand side in (C.13). This solution can be found numerically.

ii) However, for $2a_2 > 3a_1 > \sqrt[3]{2}a_2$ since $\frac{\partial^2 p^e_{ppu}(w)}{\partial w^2} > 0$, the slope of right hand side is < 1 and may even be < 0. Therefore, there may be multiple solutions to (C.12) when $2a_2 > 3a_1 > \sqrt[3]{2}a_2$ which refers to different equilibrium wholesale prices w^e_2.

iii) This result follows from the equilibrium profit of the pay-per-unit distributor when $a_1 > \frac{2a_2}{3}$. □

Proof of Proposition 16 for Case 3:

Proof. The pay-per-unit distributor sets the wholesale price such that

$$w_3^e = \arg\max_{w\geq 0}(p_{ppu}^e(w) - w)D_{ppu}(p_{ppu}^e, p_{sub}^e) = \arg\max_{w\geq 0} \frac{N(p_{ppu}^e(w) - w)(8a_2^3 - 27a_1^3)}{324(a_2 - a_1)(c + p_{ppu}^e(w))^2}.$$

The w_3^e that maximizes her profit must satisfy the first order condition:

$$w = \frac{p_{ppu}^e(w) - c}{2} + \frac{c + p_{ppu}^e(w)}{2\frac{\partial p_{ppu}^e(w)}{\partial w}}. \tag{C.14}$$

Proof for unique solution: Left hand side in above equation is increasing with a slope 1. taking derivative of the right-hand side w.r.t. w for $\sqrt[3]{2}a_2 > 3a_1$x, we get

$$1 + \frac{\partial p_{ppu}^e(w)}{\partial w} - \frac{c + p_{ppu}^e(w)}{\left(\frac{\partial p_{ppu}^e(w)}{\partial w}\right)^2}\frac{\partial^2 p_{ppu}^e(w)}{\partial w^2} > 1 - \frac{c + p_{ppu}^e(w)}{\left(\frac{\partial p_{ppu}^e(w)}{\partial w}\right)^2}\frac{\partial^2 p_{ppu}^e(w)}{\partial w^2} > 1. \tag{C.15}$$

Both left hand side and right hand side of (C.14) are increasing in w and we know that there exists a solution w_3^e for $\sqrt[3]{2}a_2 > 3a_1$. This solution to (C.14) is unique , since the right hand side is increasing faster than the left hand side as seen from (C.15). This also shows that $w_3^e < w_2^e$ since the slope for right-hand side in case 3 is more than slope in case 2. However, as in Case 2, there may be multiple solutions to (C.14) when $2a_2 > 3a_1 > \sqrt[3]{2}a_2$, which refers to the different equilibrium wholesale prices w_3^e. □

† Weak Entrant and Weak Incumbent with Pay-per-unit Pricing: Before the entry of a weak pay-per-unit pricing, the incumbent earns $N(a_2^2 + a_2a_1 + a_1^2)/96c$ by charging $p_{ppu}^* = 3c$ and the content provider sets the wholesale price contract $w^* = c$. When the weak entrant chooses pay-per-unit pricing modality the two distributors will end in a classical price war due to perfect competition if content provider sets the same wholesale price contract

w with both distributors. Under perfect competition, the distributors make zero profit by charging $p_{ppu} = w$. Note that setting $w = w^*$ for both distributors will yield the content provider, same profit as before the entry of another weak pay-per-unit distributor. Consumer demand can be arbitrarily split among the distributors in that equilibrium.

When either distributor has the cost advantage due to a lower wholesale price w, she takes all the market by setting the price a cent below her competitor or w^* whichever is lower. Since content provider makes the maximum revenue by setting $w = w^*$ for a monopoly distributor, she will set $w = w^*$ for the distributor with the cost advantage and set the wholesale price for the other distributor at $w > w^*$. Therefore, $(w_1^*, w_2^*) = (w^*, w^* + \theta)$, for all $\theta \geq 0$ are equilibria. This gives us multiple equilibrium solutions where the profit of the cost advantage distributor is the same as monopolist $N(a_2^2 + a_2 a_1 + a_1^2)/96c$, however it is unclear if the entrant of the incumbent gets the cost advantage. Although a weak subscription pricing distributor may generate positive revenues, all of the revenues are extracted by a strong content provider dictating the licensing contract. There is no clear dominant pricing strategy in such a setting, and there are multiple subgame perfect Nash equilibria.

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